

UNIVERSITÀ DEGLI STUDI DI BRESCIA
Dipartimento di Ingegneria dell'Informazione



Data-Based Optimization for Applications to Decision-Making, Identification and Control

A Study of Coverage Properties

Dottorando: **Algo Carè**

Relatore: **Prof. Marco C. Campi**

Coordinatore

Correlatore: **Ing. Simone Garatti**

del dottorato: **Prof. Alfonso Gerevini**

XXV CICLO DI DOTTORATO DI RICERCA IN
INGEGNERIA INFORMATICA ED AUTOMATICA

Schema della presentazione

I. L'approccio a scenario

II. Analisi teorica

Introduzione all'approccio a scenario

I. L'approccio a scenario

II. Analisi teorica

Un problema convesso

$$\min_{\gamma} \ell(\gamma)$$

Un problema convesso

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$$\gamma \in \Gamma \subseteq \mathbb{R}^d$$

Un problema convesso

$$\min_{\gamma} \ell(\gamma)$$

$$\gamma \in \Gamma \subseteq \mathbb{R}^d$$

$$\ell(\gamma) : \Gamma \rightarrow \mathbb{R} \quad \text{convessa}$$

Interpretazione

$$\min_{\gamma} \ell(\gamma)$$

$$\gamma \in \Gamma \subseteq \mathbb{R}^d$$

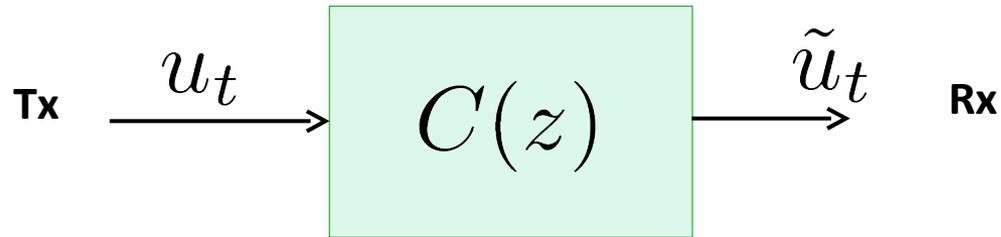
parametri di progetto

$$\ell(\gamma) : \Gamma \rightarrow \mathbb{R}$$

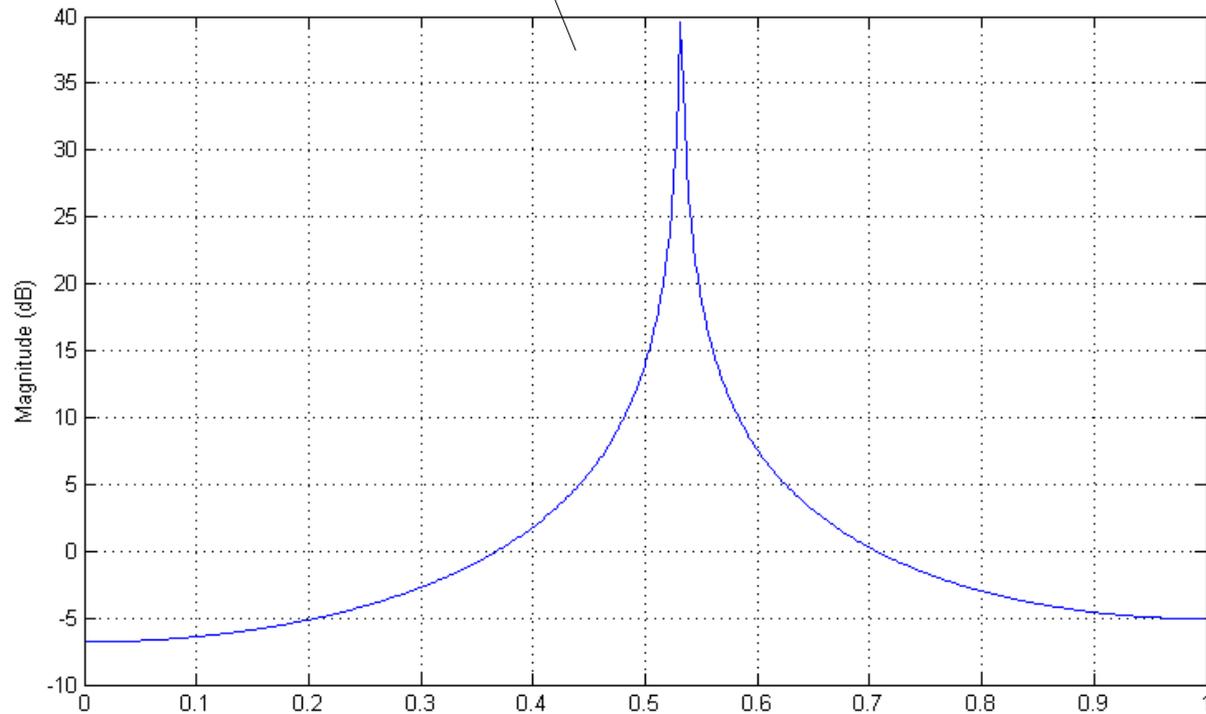
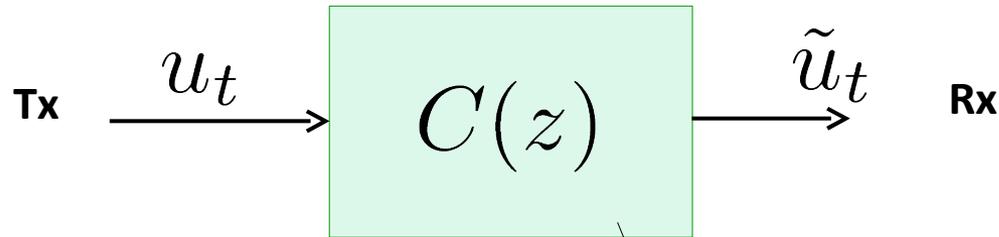
convessa

costo

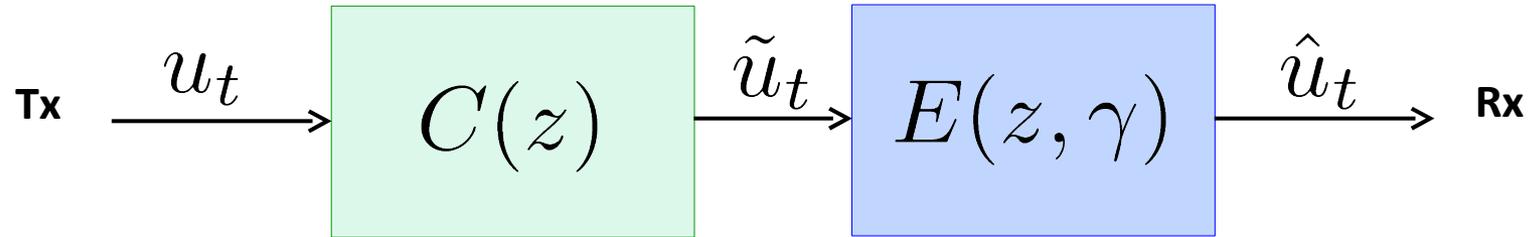
Es. - Equalizzazione di canale



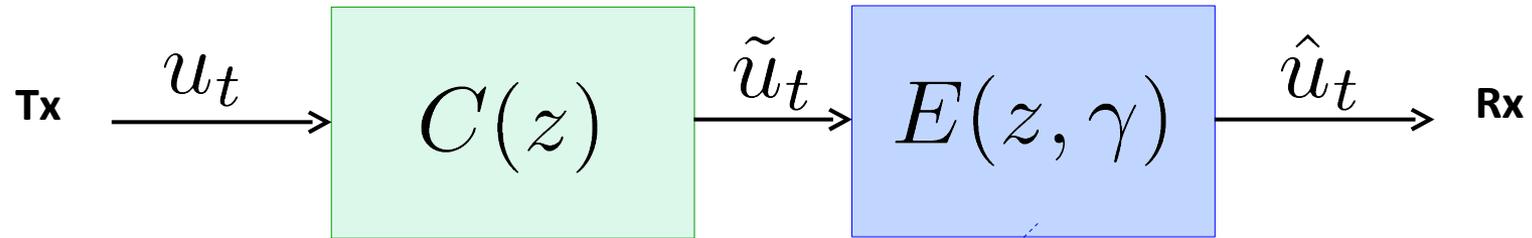
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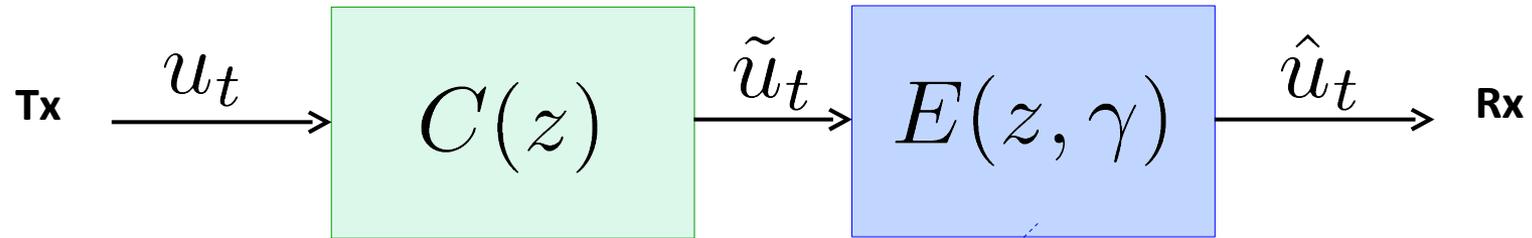


Es. - Equalizzazione di canale



$$E(z, \gamma) = \gamma_1 + \gamma_2 z^{-1} + \cdots + \gamma_d z^{-d+1}$$

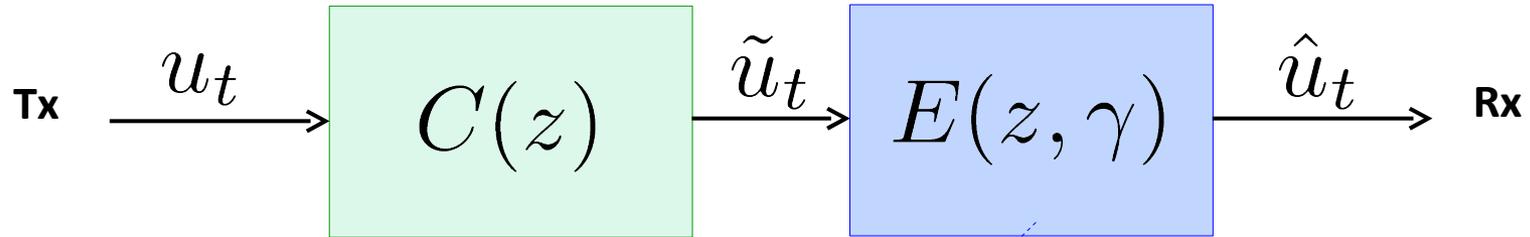
Es. - Equalizzazione di canale



$$E(z, \gamma) = \gamma_1 + \gamma_2 z^{-1} + \cdots + \gamma_d z^{-d+1}$$

$$C(z)E(z, \gamma) \approx z^{-D}$$

Es. - Equalizzazione di canale

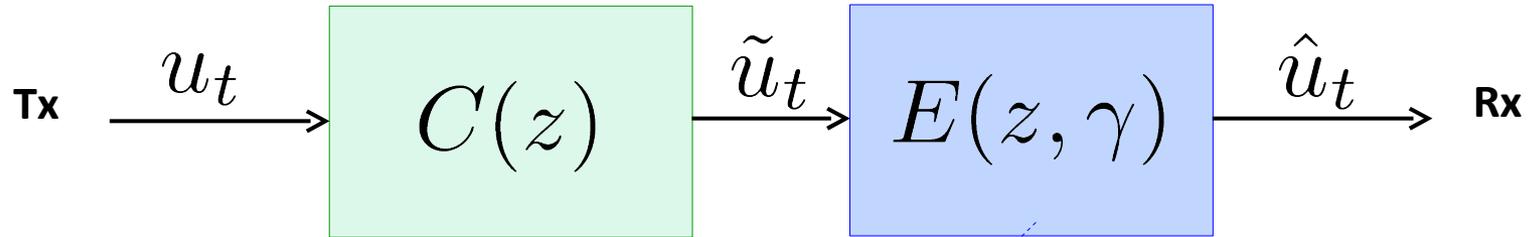


$$E(z, \gamma) = \gamma_1 + \gamma_2 z^{-1} + \cdots + \gamma_d z^{-d+1}$$

$$C(z)E(z, \gamma) \approx z^{-D}$$

$$\ell(\gamma) = \|C(z)E(z, \gamma) - z^{-D}\|_{\infty}$$

Es. - Equalizzazione di canale



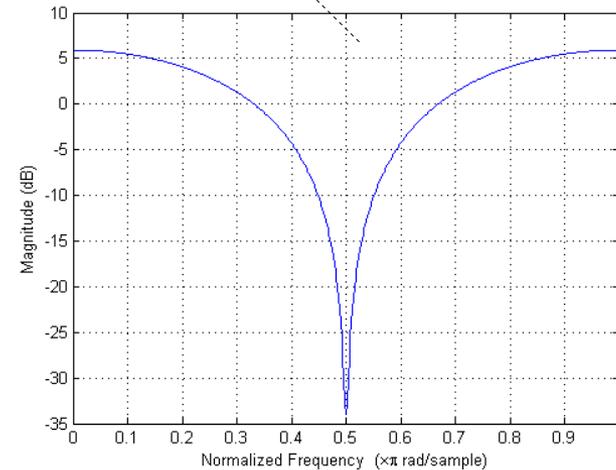
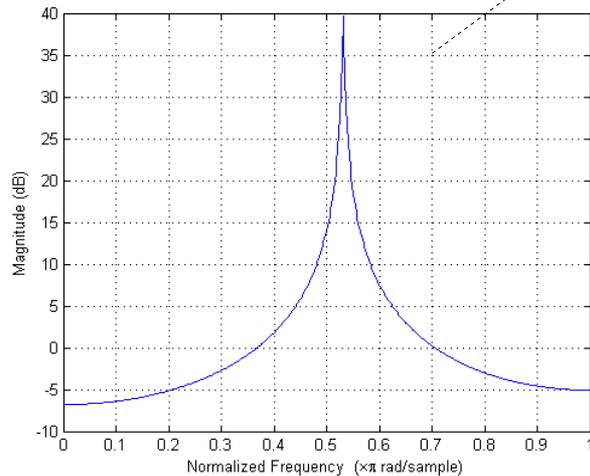
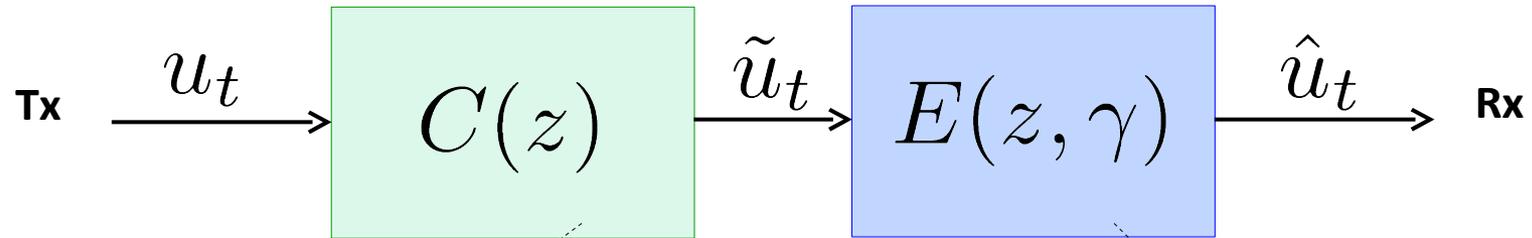
$$E(z, \gamma) = \gamma_1 + \gamma_2 z^{-1} + \cdots + \gamma_d z^{-d+1}$$

$$C(z)E(z, \gamma) \approx z^{-D}$$

$$\ell(\gamma) = \|C(z)E(z, \gamma) - z^{-D}\|_{\infty}$$

$$\min_{\gamma} \ell(\gamma)$$

Es. - Equalizzazione di canale



$$l(\gamma) = \|C(z)E(z, \gamma) - z^{-D}\|_{\infty}$$

$$\min_{\gamma} l(\gamma)$$

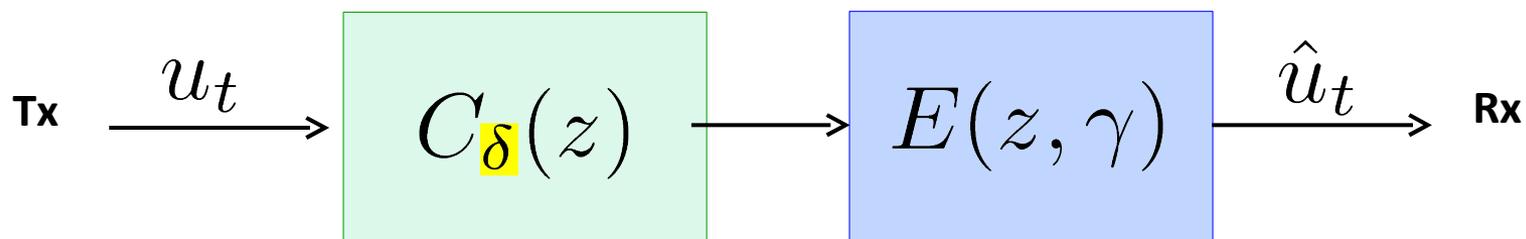
Nella vita reale

$$\min_{\gamma} l_{\delta}(\gamma)$$

Parametro incerto

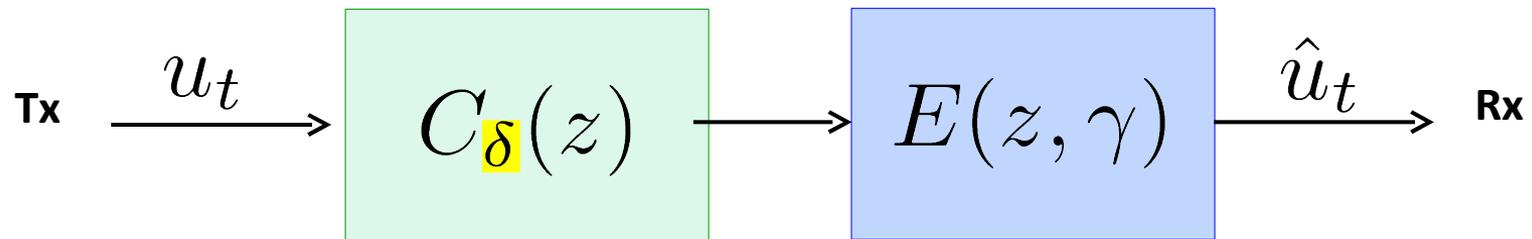
Es. Equalizzazione di canale

$$\min_{\gamma} \ell_{\delta}(\gamma)$$



Es. Equalizzazione di canale

$$\min_{\gamma} l_{\delta}(\gamma)$$



$$l_{\delta}(\gamma) = \|C_{\delta}(z)E(z, \gamma) - z^{-D}\|_{\infty}$$

Fidarsi dell'esperienza

“ Quello che mi piace dell'esperienza
è che si tratta di una cosa tanto onesta...

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potete aver ingannato voi stessi,
ma l'esperienza non sta ingannando voi.

Fidarsi dell'esperienza

“ Quello che mi piace dell'esperienza
è che si tratta di una cosa tanto onesta...
potete aver ingannato voi stessi,
ma l'esperienza non sta ingannando voi.
L'universo risponde il vero quando lo interrogate onestamente ”

C. S. Lewis.

Algoritmo a scenario

Osservazioni (“scenari”):

$$\delta^{(1)}, \delta^{(2)}, \dots, \delta^{(N)}$$

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$$l_{\delta^{(1)}}(\gamma), l_{\delta^{(2)}}(\gamma), \dots, l_{\delta^{(N)}}(\gamma)$$

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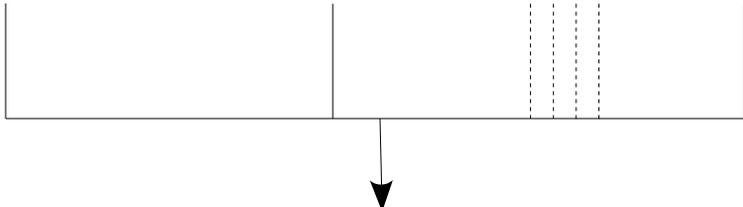


$$\frac{1}{N} \sum_{i=1}^N l_{\delta^{(i)}}(\gamma)$$

Algoritmo a scenario

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$$\delta^{(1)}, \delta^{(2)}, \dots, \delta^{(N)}$$

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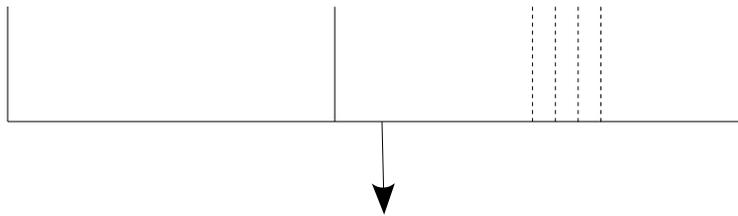
$$\min_{\gamma} \frac{1}{N} \sum_{i=1}^N l_{\delta^{(i)}}(\gamma)$$

Algoritmo a scenario

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$$l_{\delta^{(1)}}(\gamma), l_{\delta^{(2)}}(\gamma), \dots, l_{\delta^{(N)}}(\gamma)$$

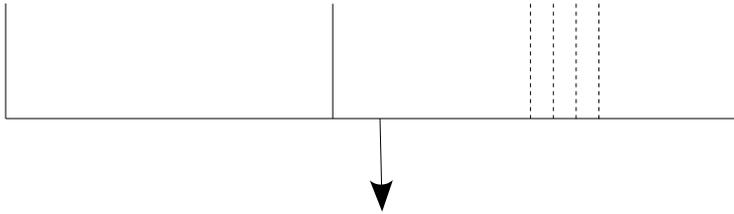


$$\max_{i=1, \dots, N} l_{\delta^{(i)}}(\gamma)$$

Algoritmo a scenario

Osservazioni (“scenari”):

$$\delta^{(1)}, \delta^{(2)}, \dots, \delta^{(N)}$$

$$l_{\delta^{(1)}}(\gamma), l_{\delta^{(2)}}(\gamma), \dots, l_{\delta^{(N)}}(\gamma)$$


$$\min_{\gamma} \max_{i=1, \dots, N} l_{\delta^{(i)}}(\gamma)$$

Algoritmo a scenario

Osservazioni (“scenari”):

$$\delta^{(1)}, \delta^{(2)}, \dots, \delta^{(N)}$$

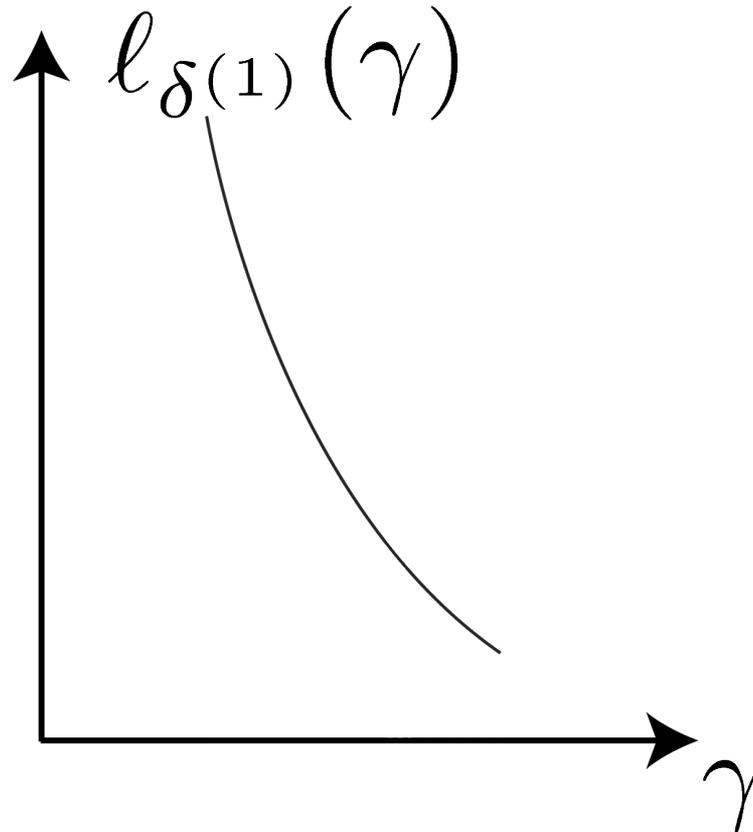
$$l_{\delta^{(1)}}(\gamma), l_{\delta^{(2)}}(\gamma), \dots, l_{\delta^{(N)}}(\gamma)$$


$$\min_{\gamma} \max_{i=1, \dots, N} l_{\delta^{(i)}}(\gamma)$$

γ^*
 progetto
 “worst-case”
 empirico

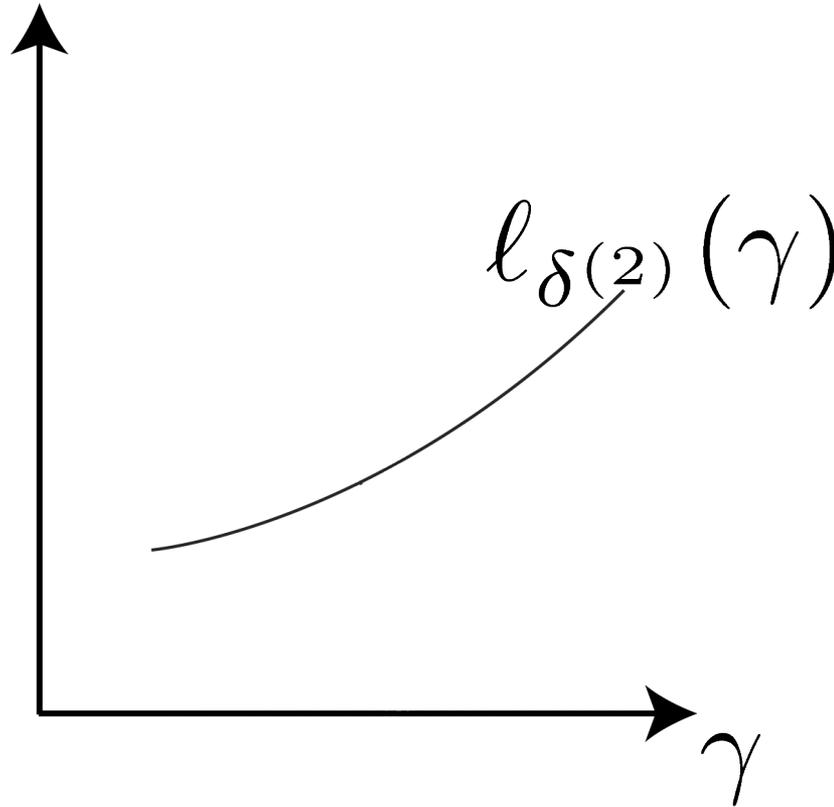
Riformulazione

$$\min_{\gamma} \max\{\ell_{\delta(1)}(\gamma), \ell_{\delta(2)}(\gamma)\}$$



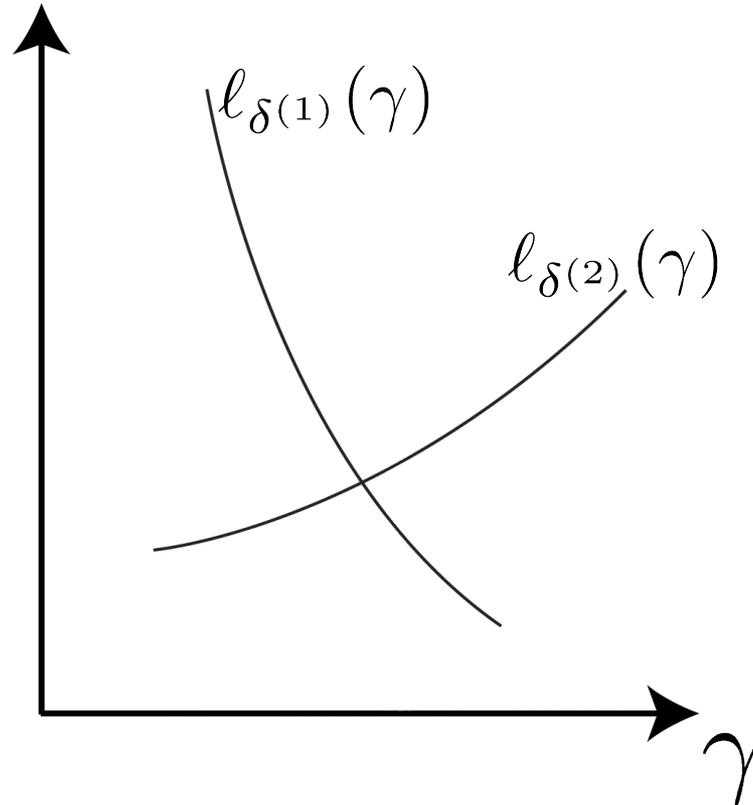
Riformulazione

$$\min_{\gamma} \max \{ \ell_{\delta(1)}(\gamma), \ell_{\delta(2)}(\gamma) \}$$



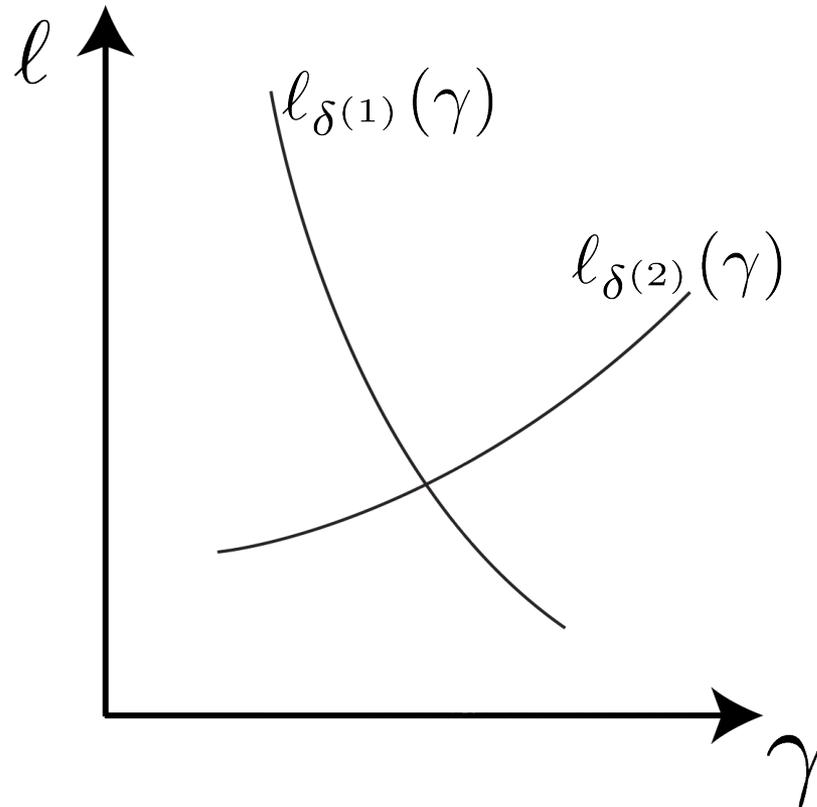
Riformulazione

$$\min_{\gamma} \max\{l_{\delta(1)}(\gamma), l_{\delta(2)}(\gamma)\}$$



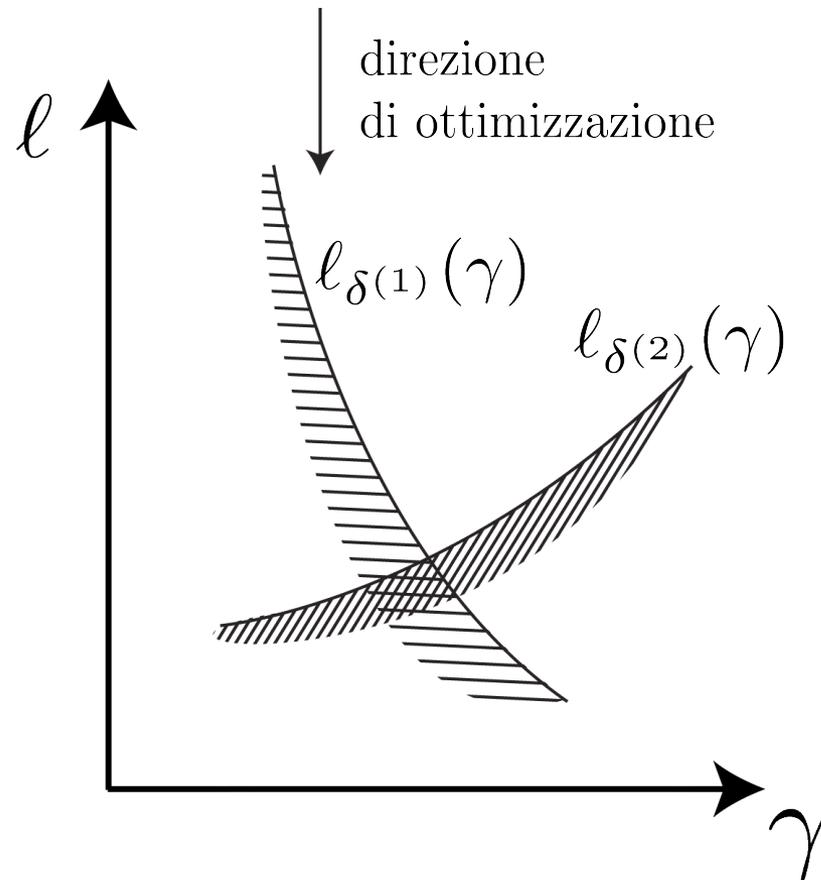
Riformulazione

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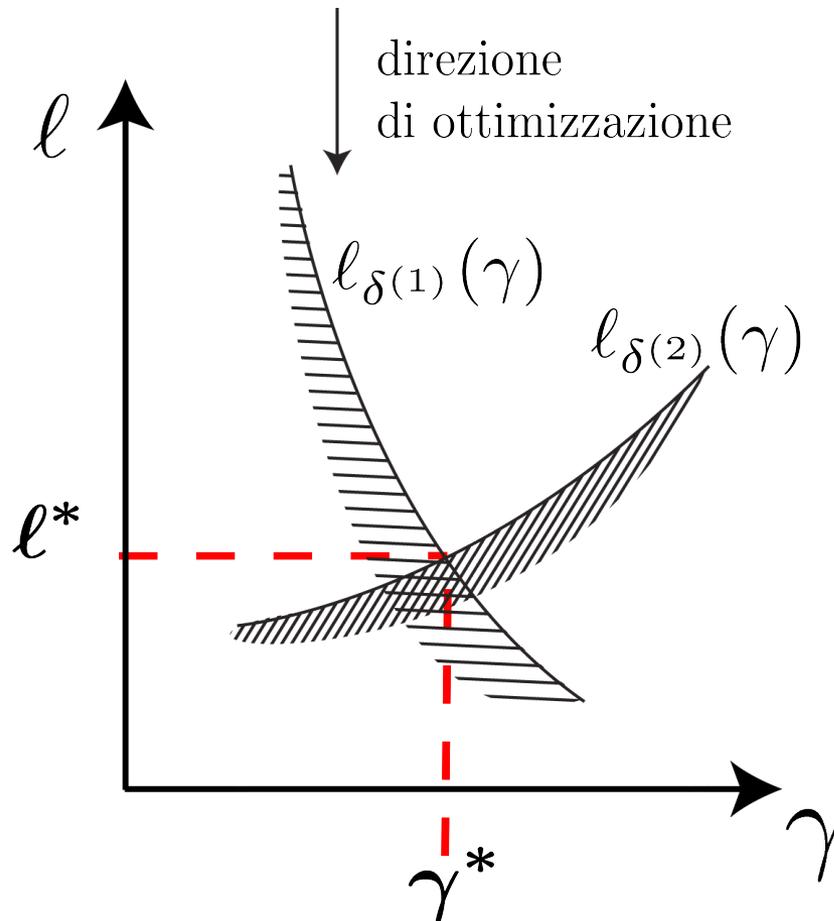
Riformulazione

$$\min_{\gamma} \max \{l_{\delta(1)}(\gamma), l_{\delta(2)}(\gamma)\}$$



Riformulazione

$$\min_{\gamma} \max \{l_{\delta(1)}(\gamma), l_{\delta(2)}(\gamma)\}$$



Riformulazione

$$\min_{\gamma} \max_{i=1, \dots, N} \ell_{\delta^{(i)}}(\gamma)$$



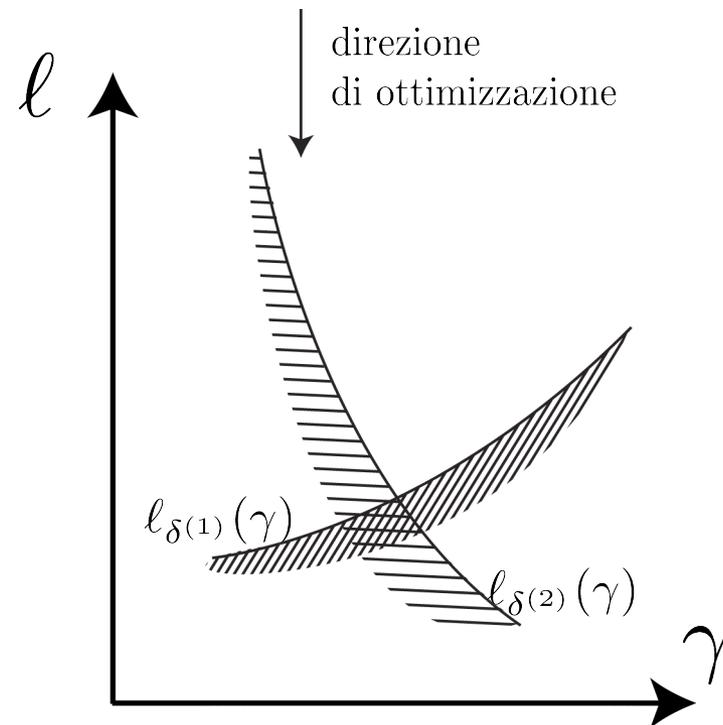
$$\min_{\gamma, \ell} \ell$$

$$\text{subject to: } \begin{aligned} \ell_{\delta^{(1)}}(\gamma) &\leq \ell, \\ \ell_{\delta^{(2)}}(\gamma) &\leq \ell, \\ &\vdots \\ \ell_{\delta^{(N)}}(\gamma) &\leq \ell. \end{aligned}$$

Risoluzione del problema a scenario

$$\min_{\gamma, l} l$$

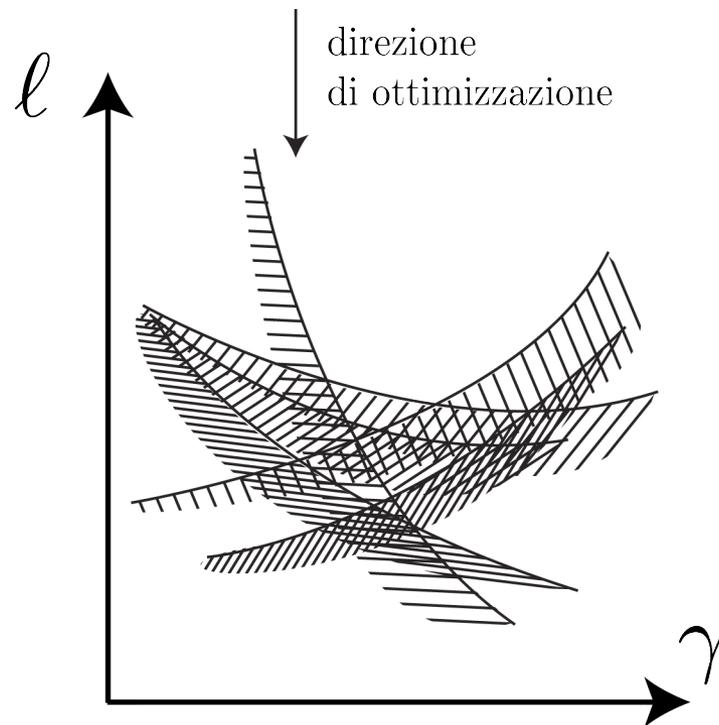
$$\text{subject to: } l_{\delta(1)}(\gamma) \leq l,$$
$$l_{\delta(2)}(\gamma) \leq l,$$



Risoluzione del problema a scenario

$$\min_{\gamma, l} l$$

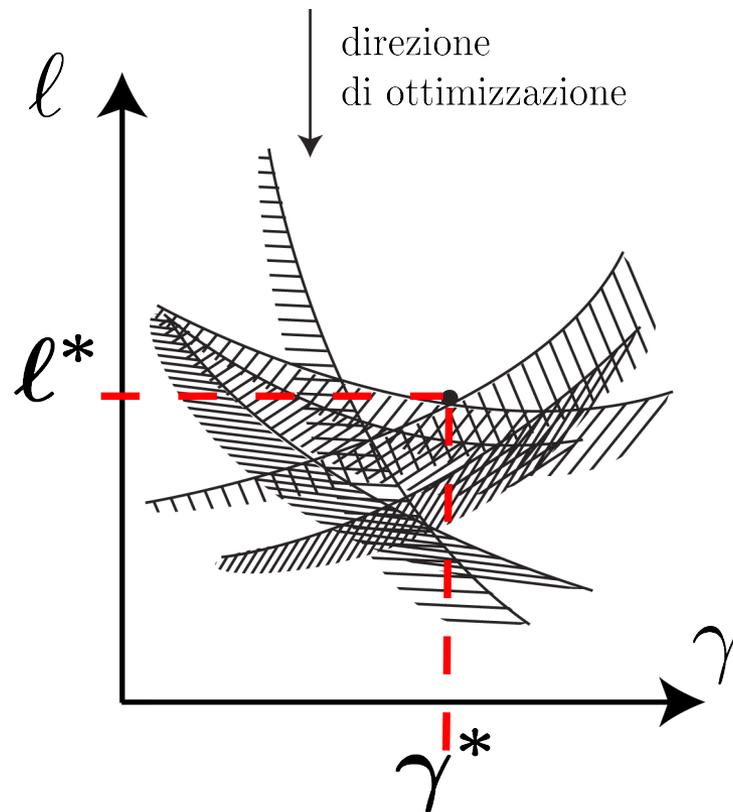
$$\text{subject to: } \begin{aligned} l_{\delta(1)}(\gamma) &\leq l, \\ l_{\delta(2)}(\gamma) &\leq l, \\ &\vdots \\ l_{\delta(N)}(\gamma) &\leq l. \end{aligned}$$



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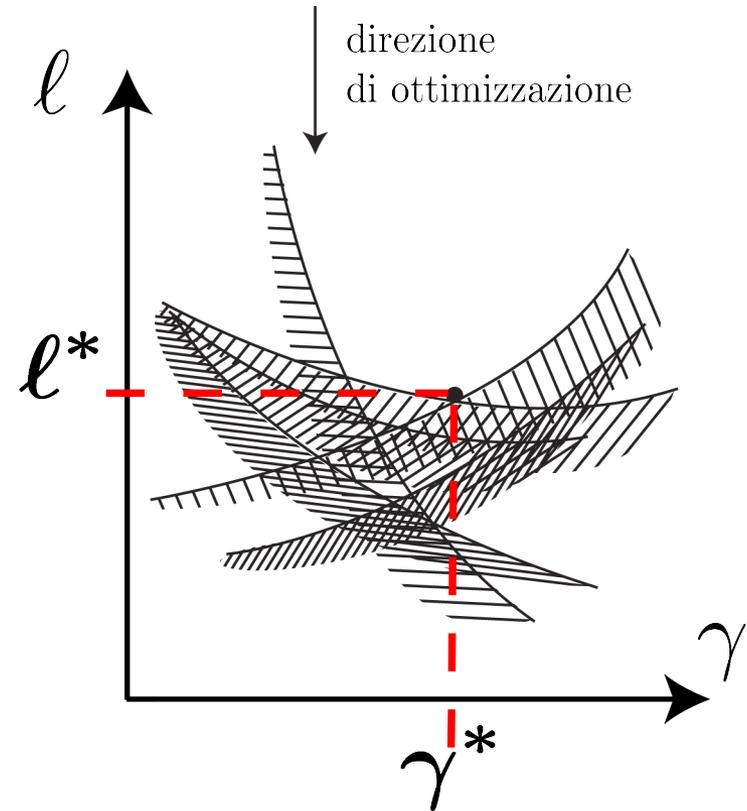


soluzione a scenario: (γ^*, l^*)

Risoluzione del problema a scenario

$$\min_{\gamma, l} l$$

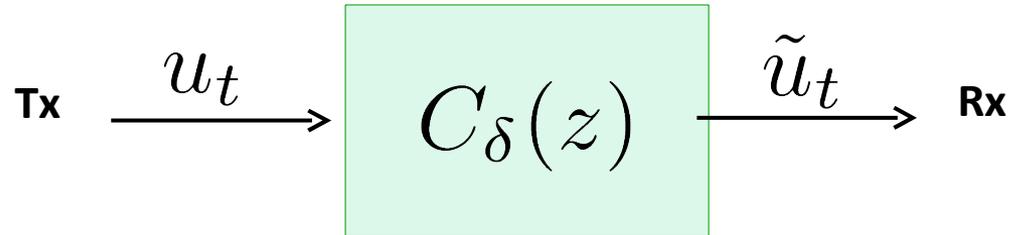
$$\text{subject to: } \begin{aligned} l_{\delta(1)}(\gamma) &\leq l, \\ l_{\delta(2)}(\gamma) &\leq l, \\ &\vdots \\ l_{\delta(N)}(\gamma) &\leq l. \end{aligned}$$



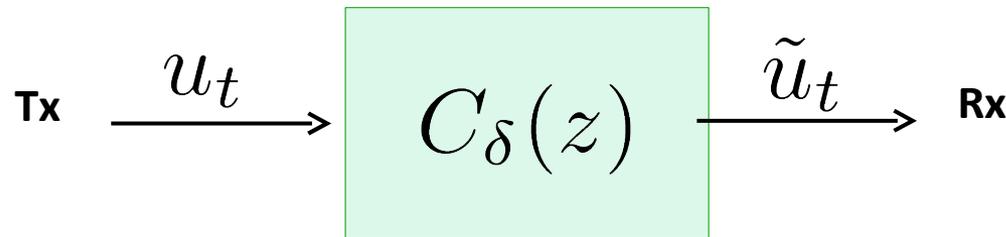
soluzione a scenario: (γ^*, l^*)

$$\max_{i=1, \dots, N} l_{\delta(i)}(\gamma^*)$$

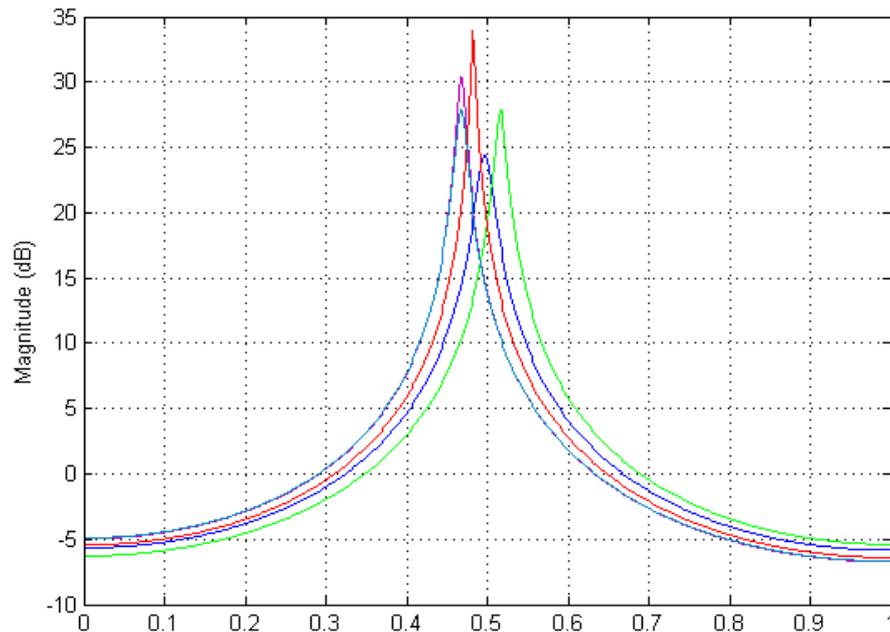
Es. - Equalizzazione di canale



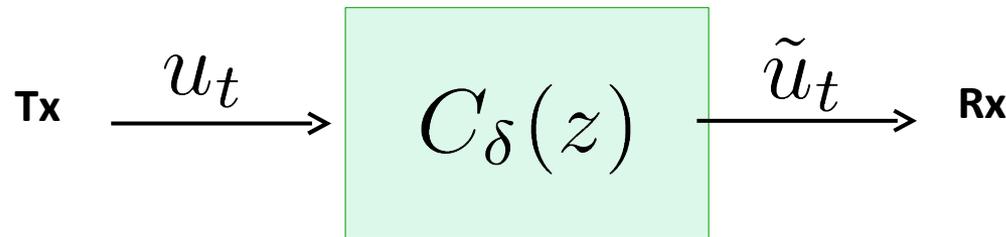
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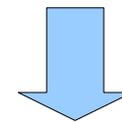
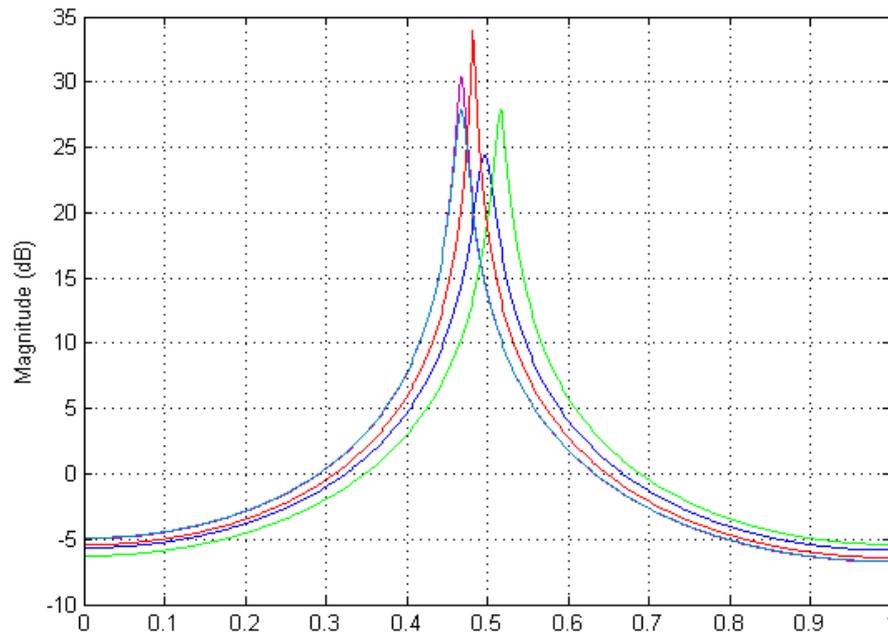
$\delta^{(1)}, \delta^{(2)}, \dots, \delta^{(N)}$



Es. - Equalizzazione di canale

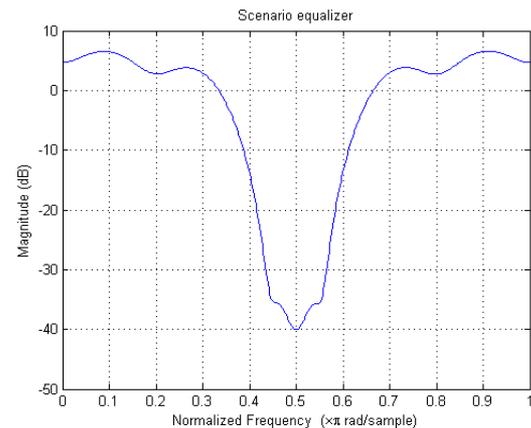
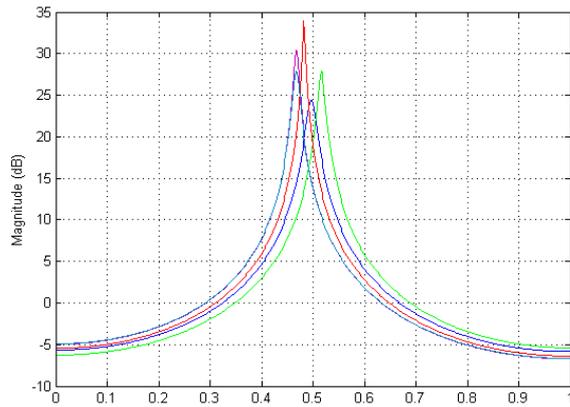
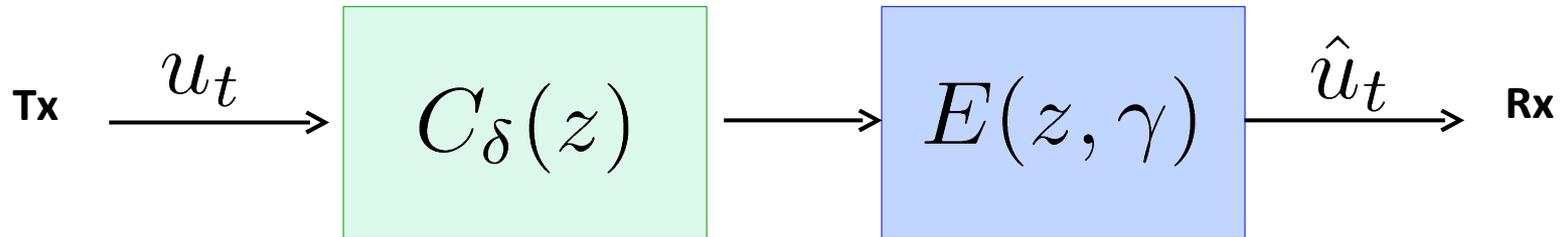


$\delta^{(1)}, \delta^{(2)}, \dots, \delta^{(N)}$



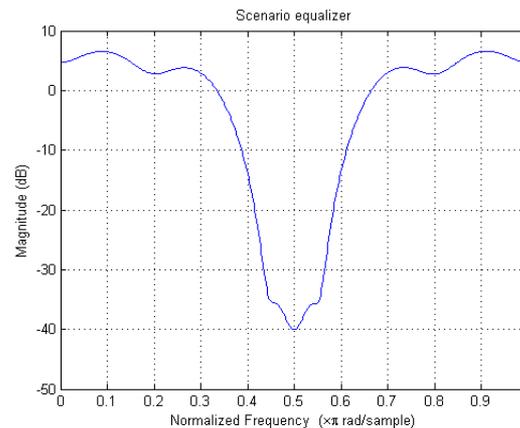
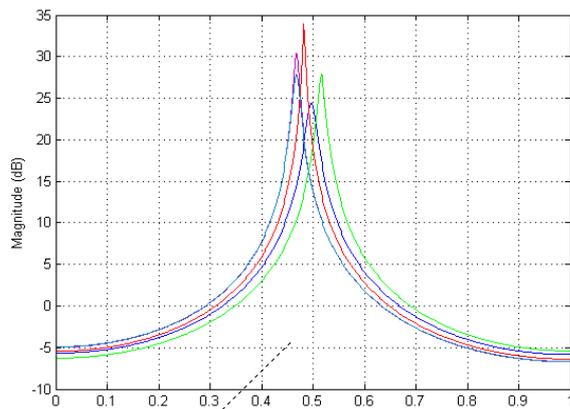
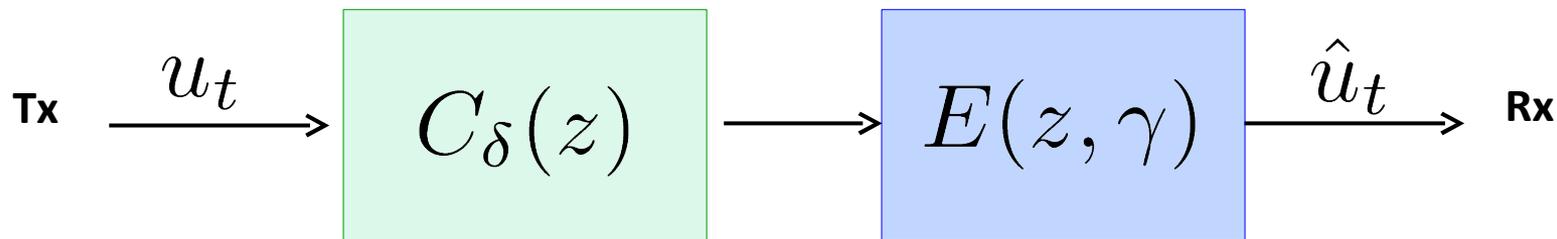
γ^*

Es. - Equalizzazione di canale



γ^*

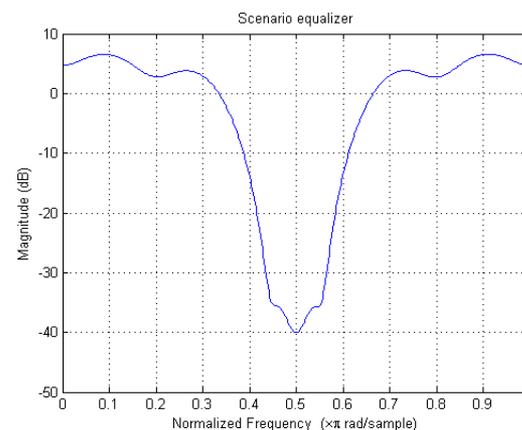
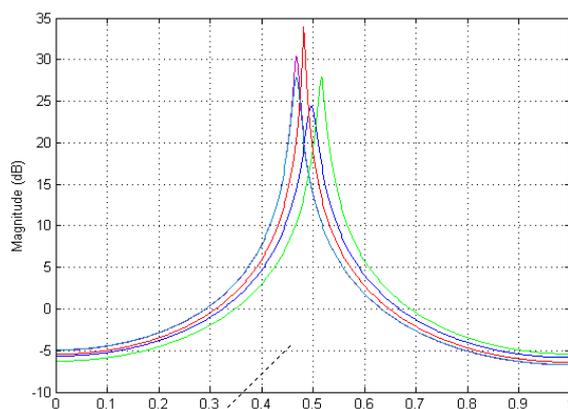
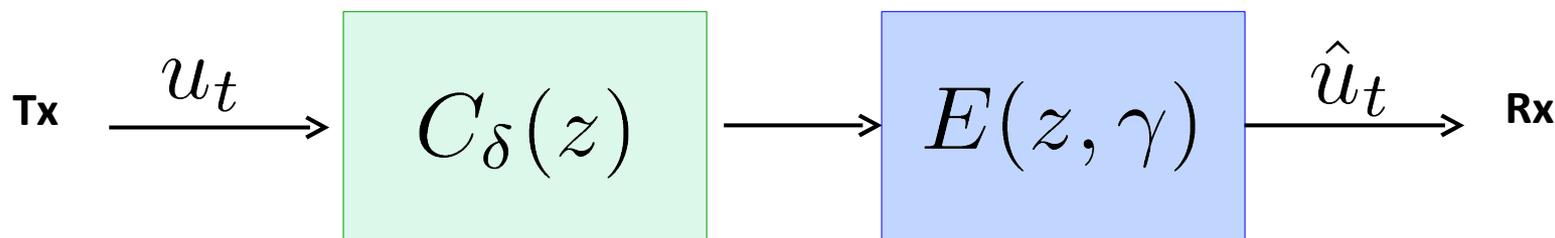
Es. - Equalizzazione di canale



γ^*

$$l_{\delta(1)}(\gamma^*), l_{\delta(2)}(\gamma^*), \dots, l_{\delta(N)}(\gamma^*)$$

Es. - Equalizzazione di canale

 γ^*

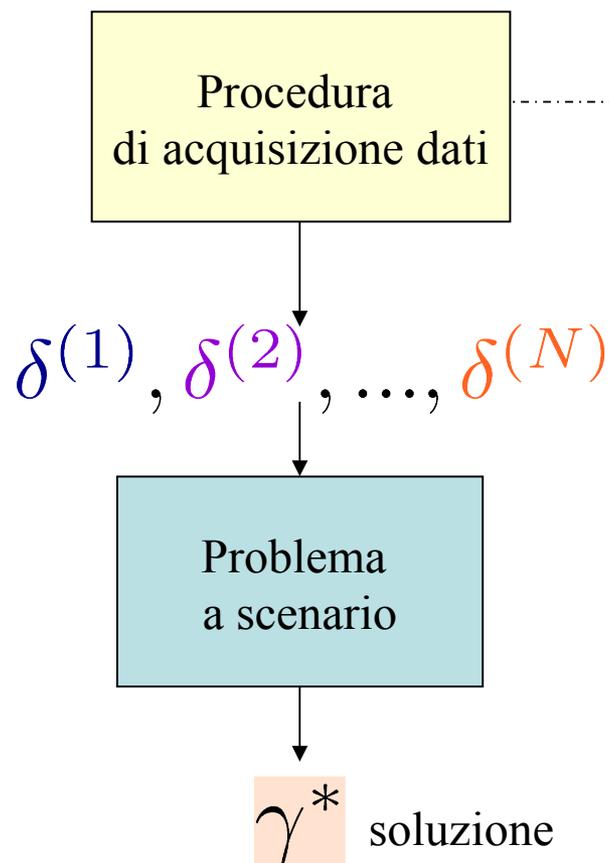
$$l_{\delta(1)}(\gamma^*), l_{\delta(2)}(\gamma^*), \dots, l_{\delta(N)}(\gamma^*)$$

$$l^* = \max_{i=1, \dots, N} l_{\delta(i)}(\gamma^*)$$

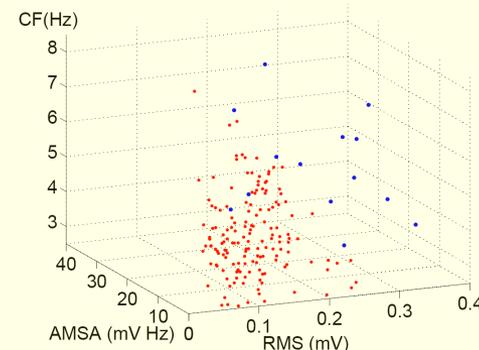
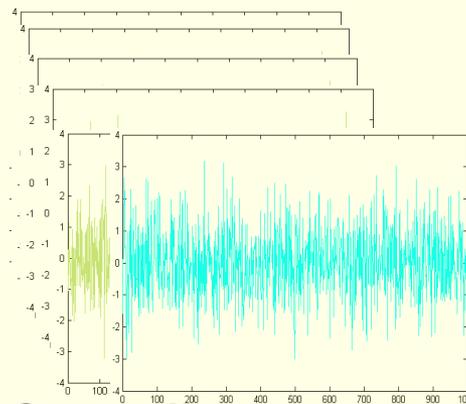
Rischio

$$l_{\delta}(\gamma^*) > l^*$$

Approccio guidato dai dati (data-based)



- *Serie storiche*
- *Esperimenti sul campo*
- ...

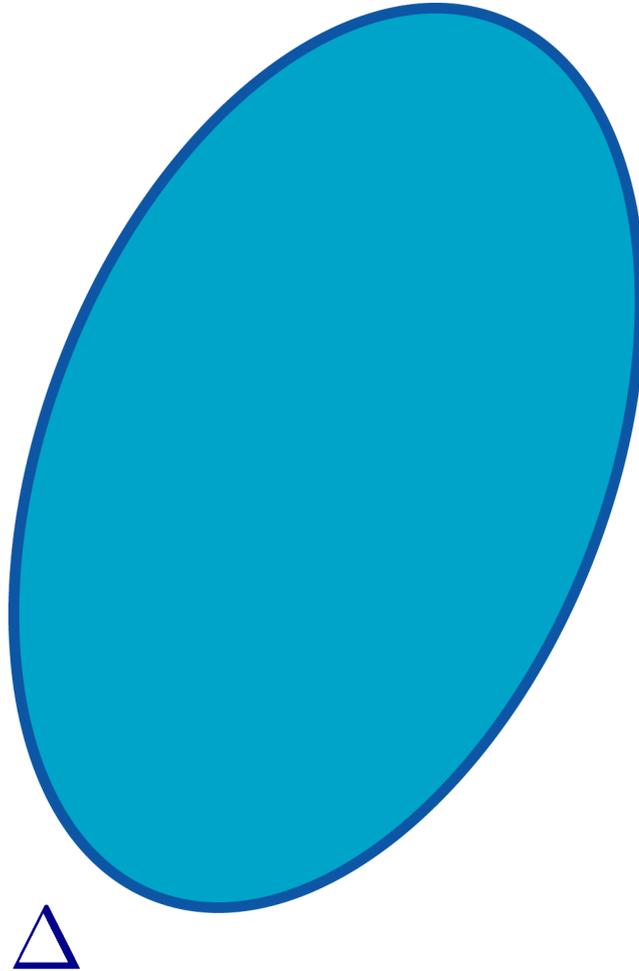


Analisi teorica

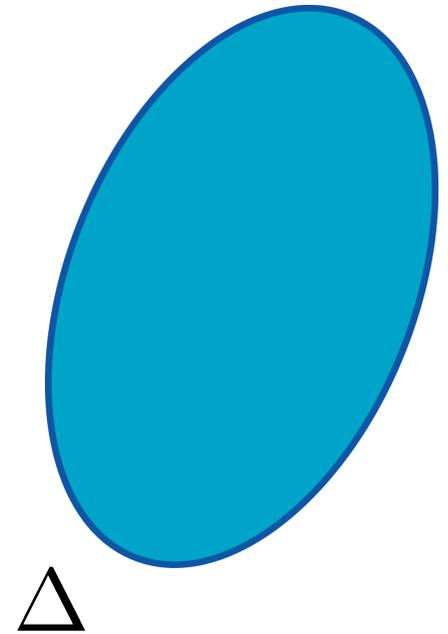
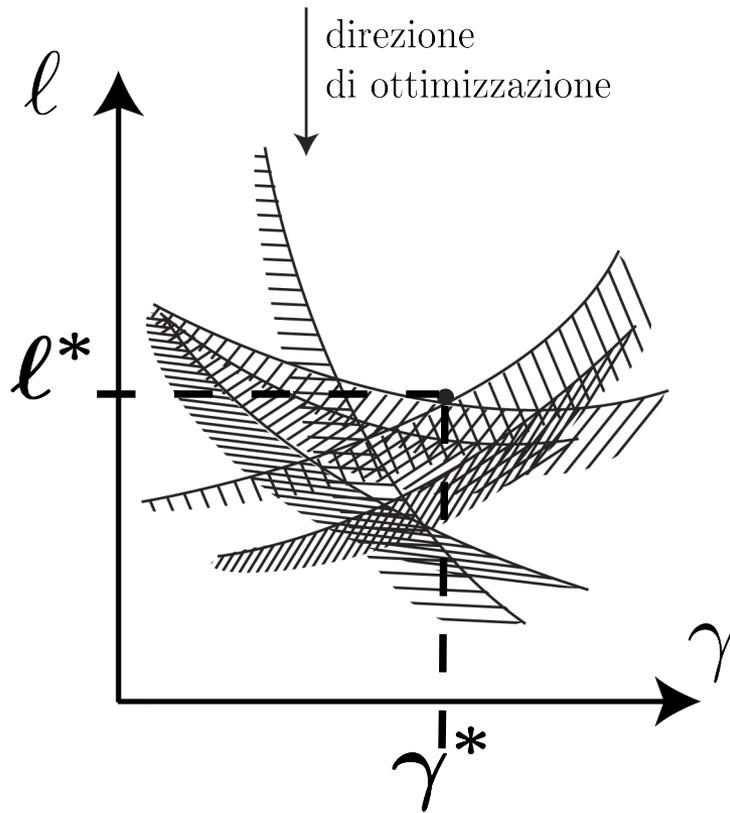
I. L'approccio a scenario

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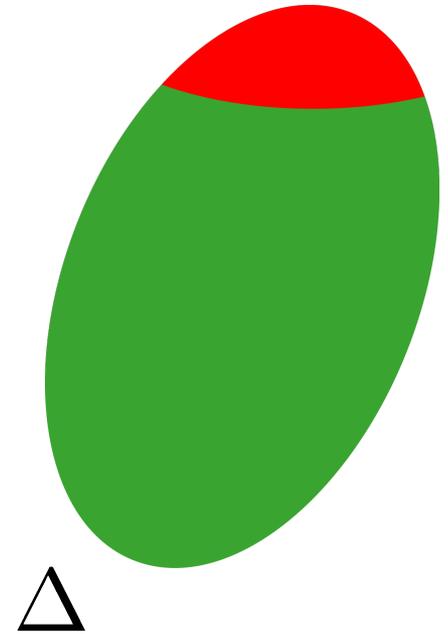
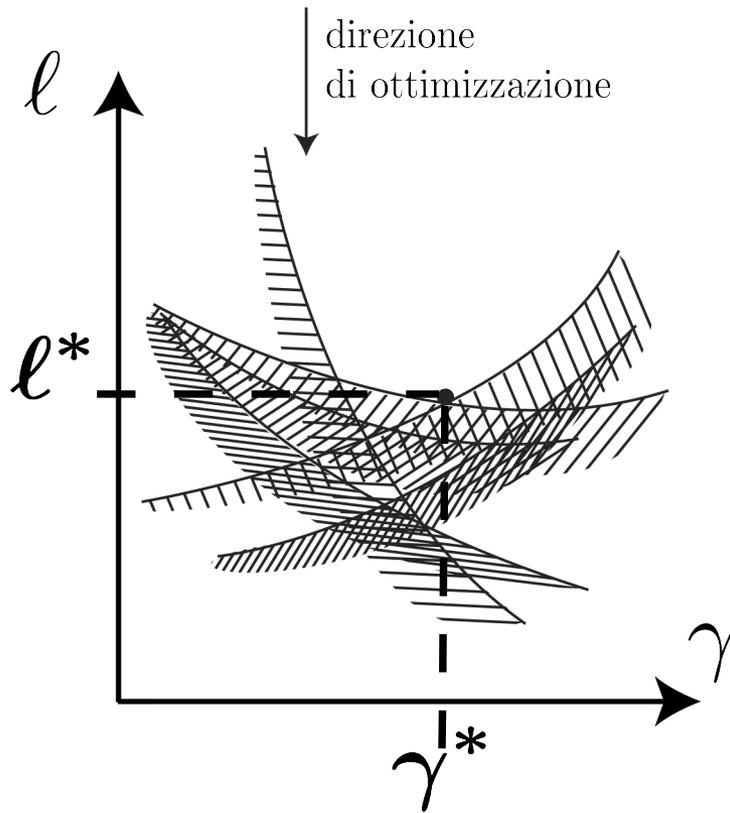
Dominio dell'incertezza



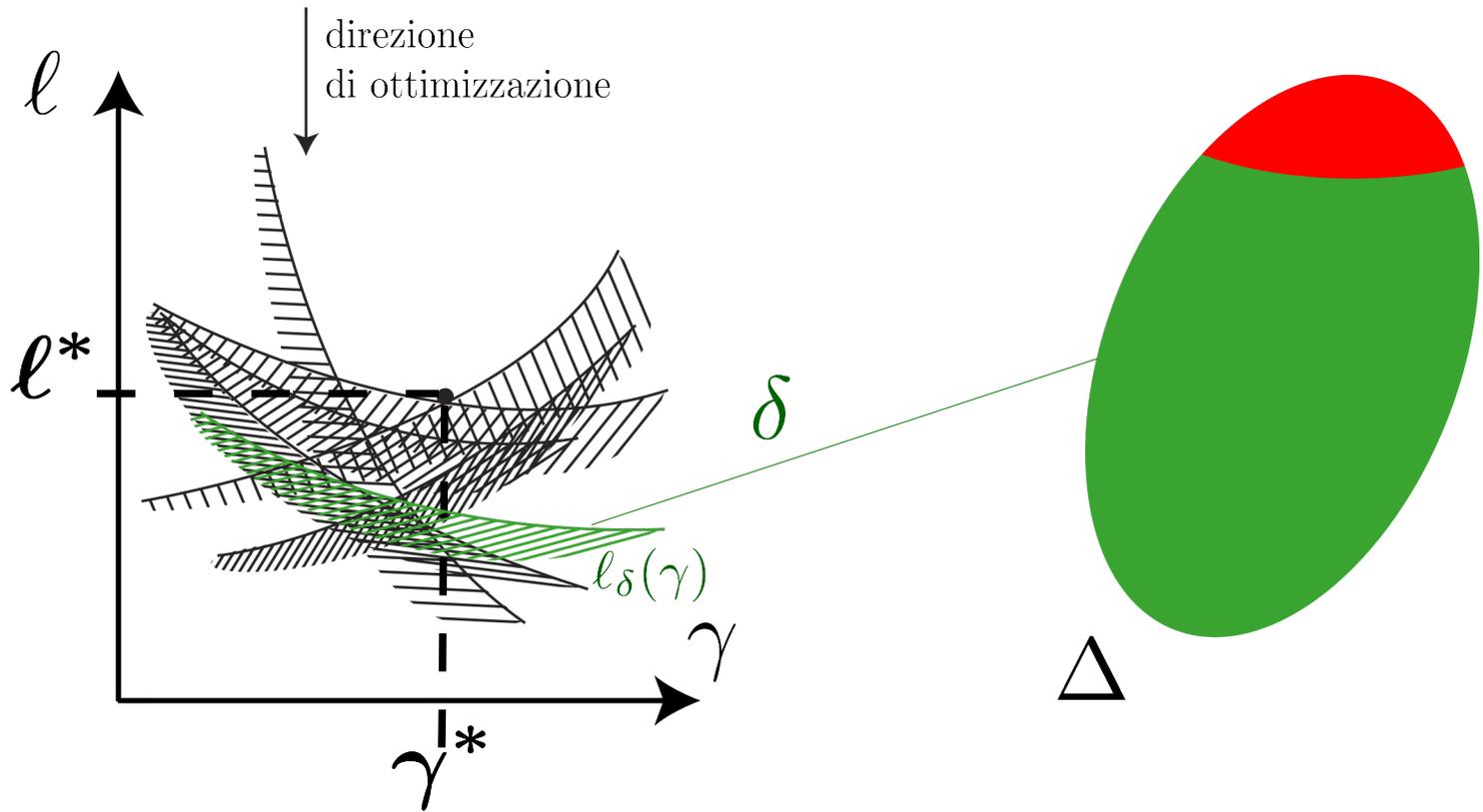
Dominio dell'incertezza



Dominio dell'incertezza

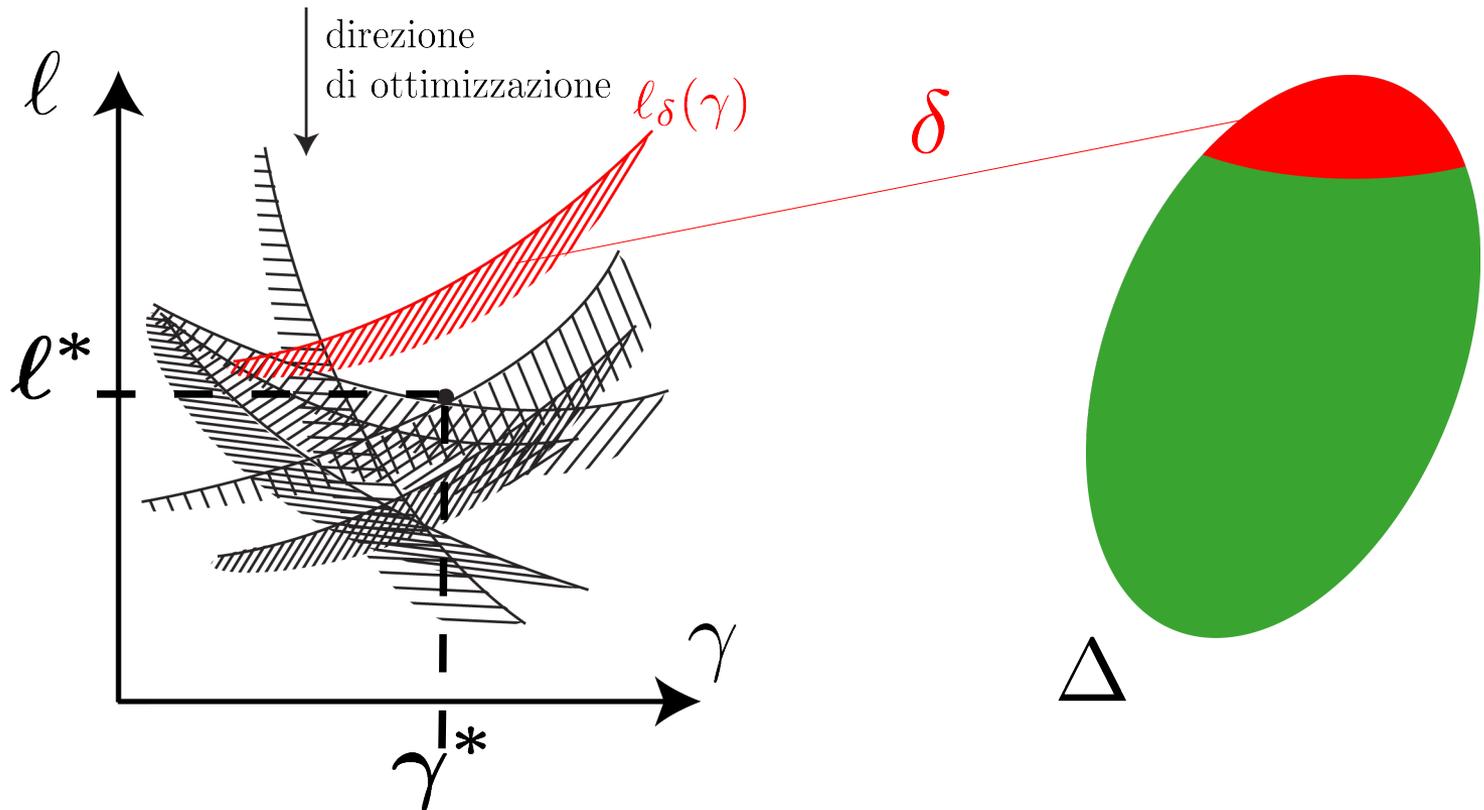


Dominio dell'incertezza



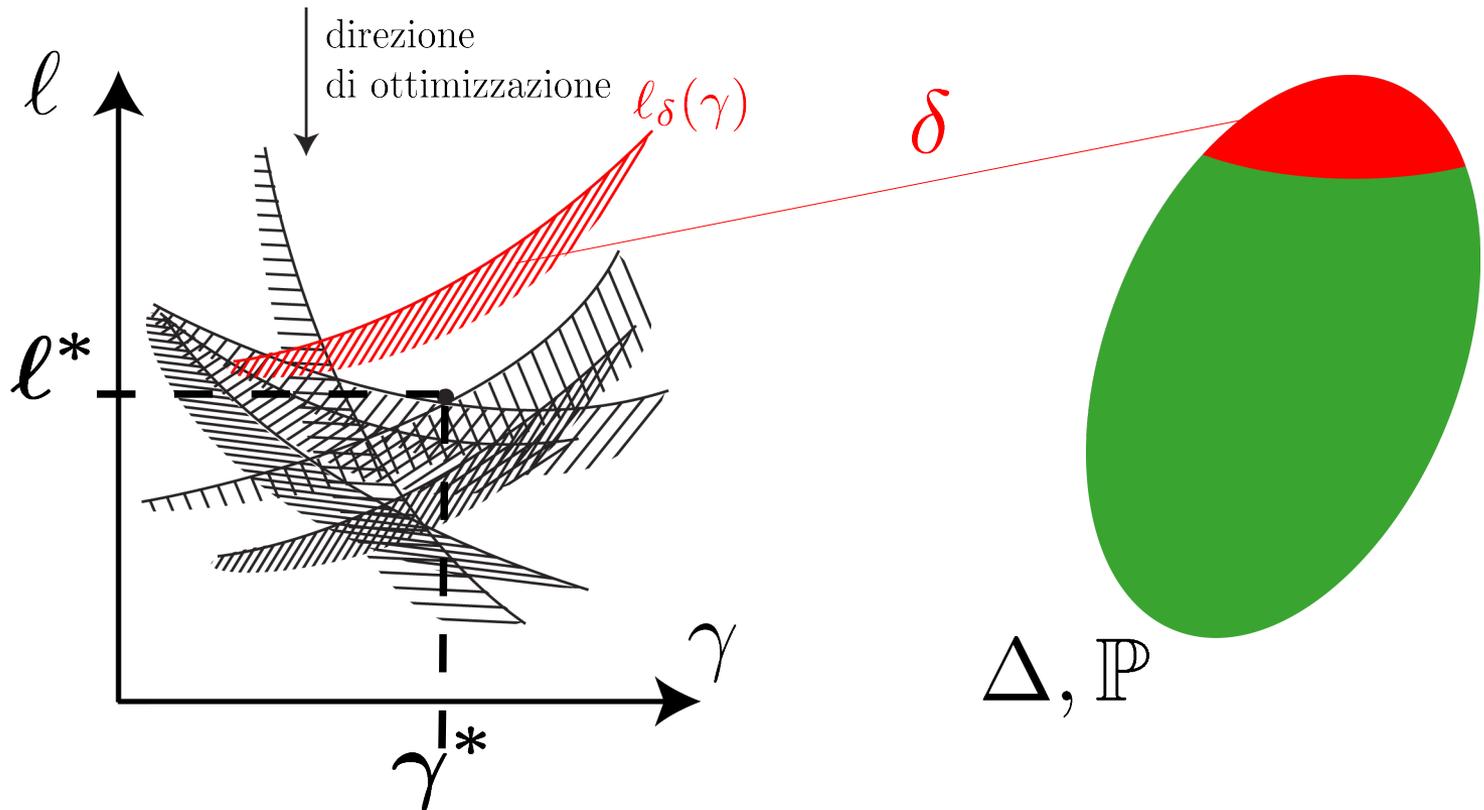
$$l_\delta(\gamma^*) \leq l^*$$

Dominio dell'incertezza



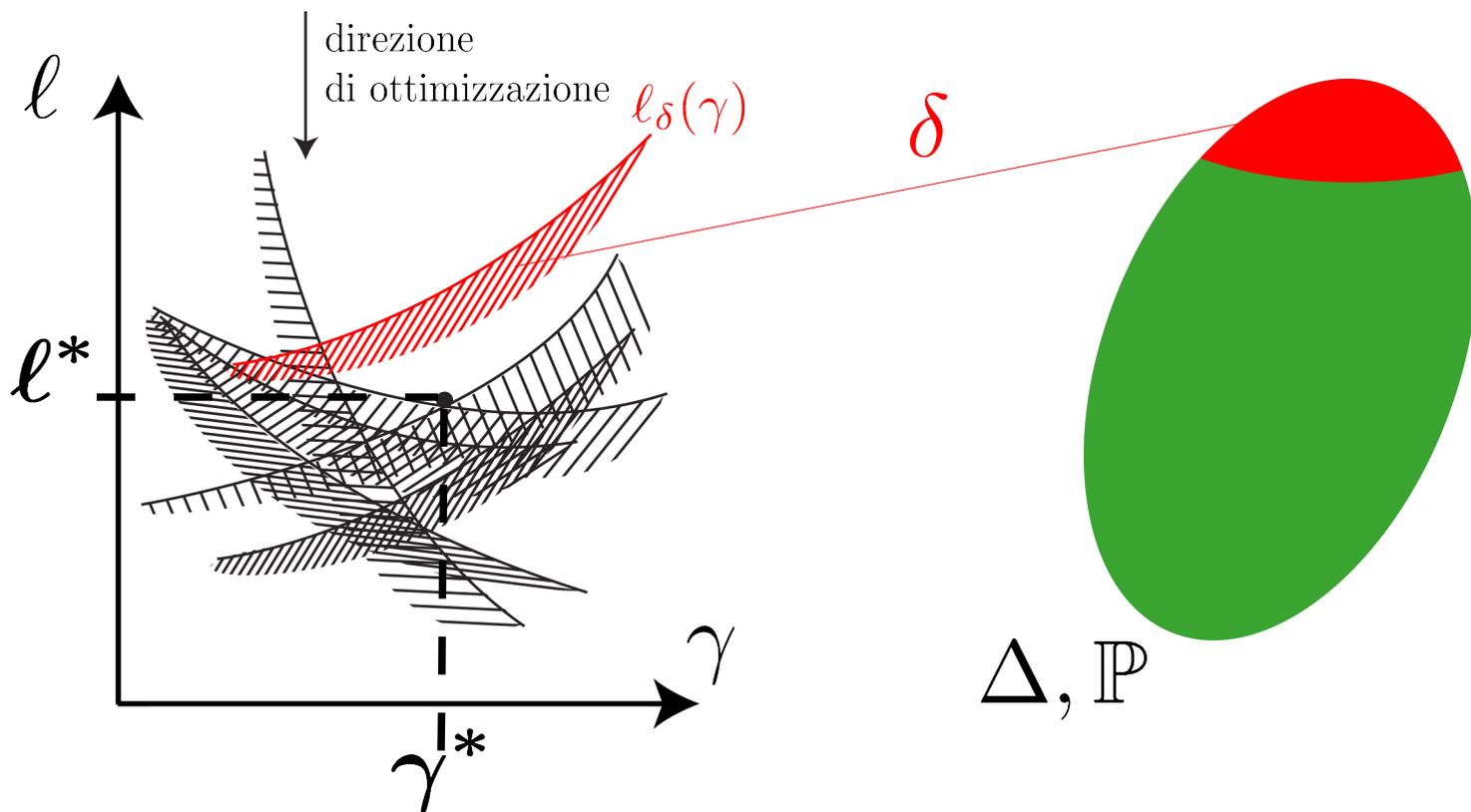
$$l_\delta(\gamma^*) > l^*$$

Dominio dell'incertezza



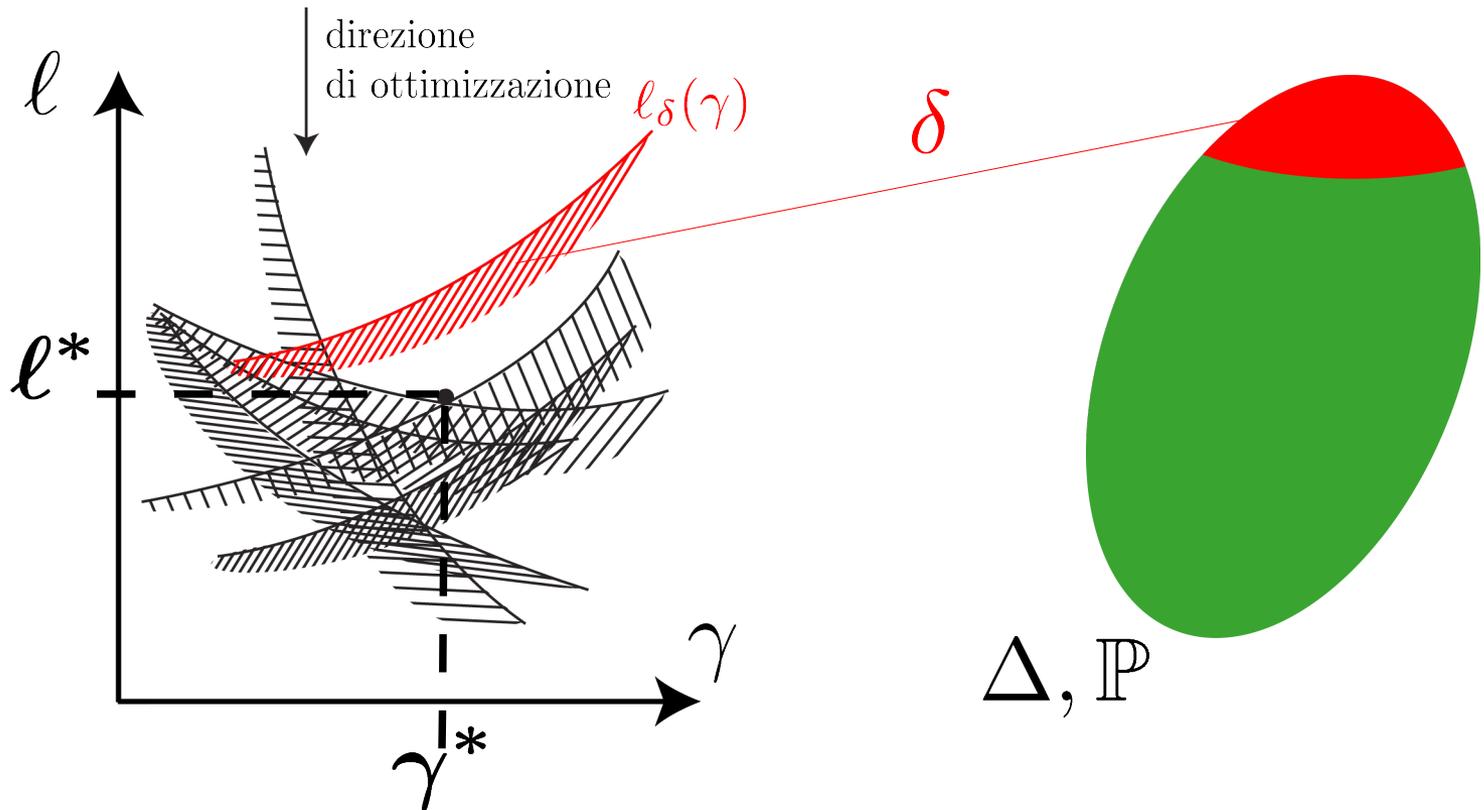
$$l_\delta(\gamma^*) > l^*$$

Rischio



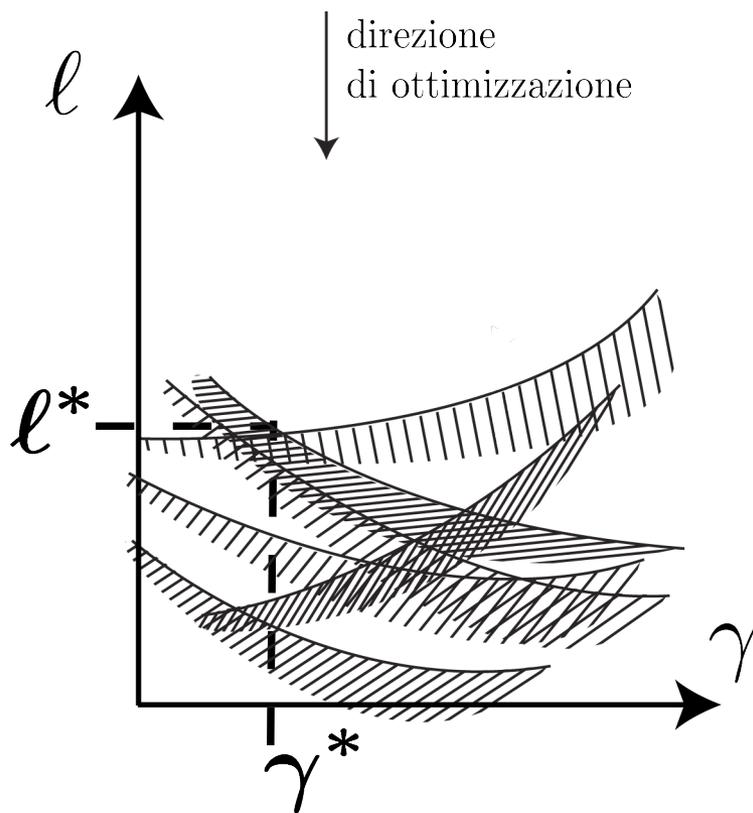
$$\mathbb{P}\{l_\delta(\gamma^*) > l^*\}$$

Rischio

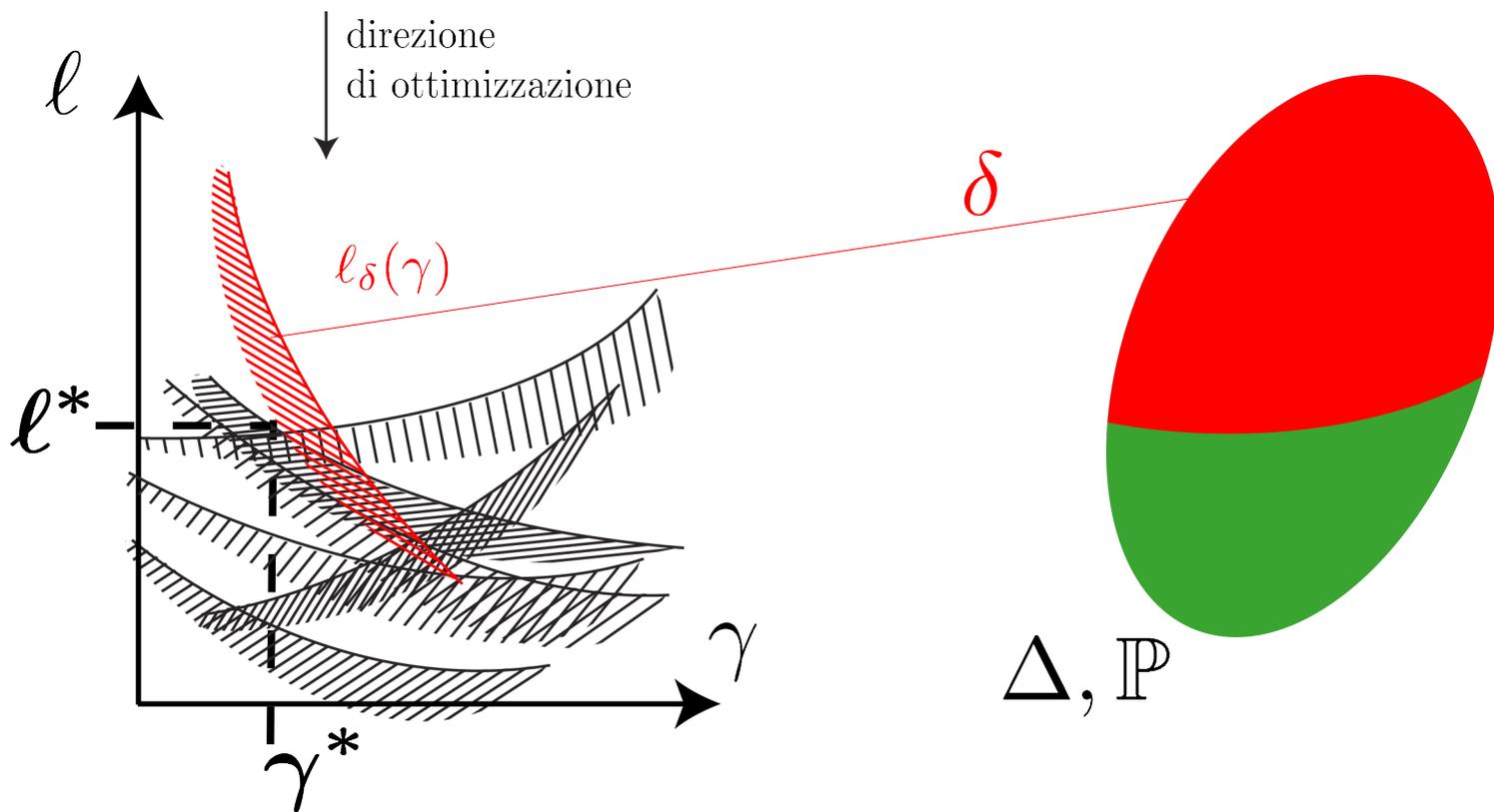


$$R = \mathbb{P}\{l_{\delta}(\gamma^*) > l^*\} \quad \text{Rischio}$$

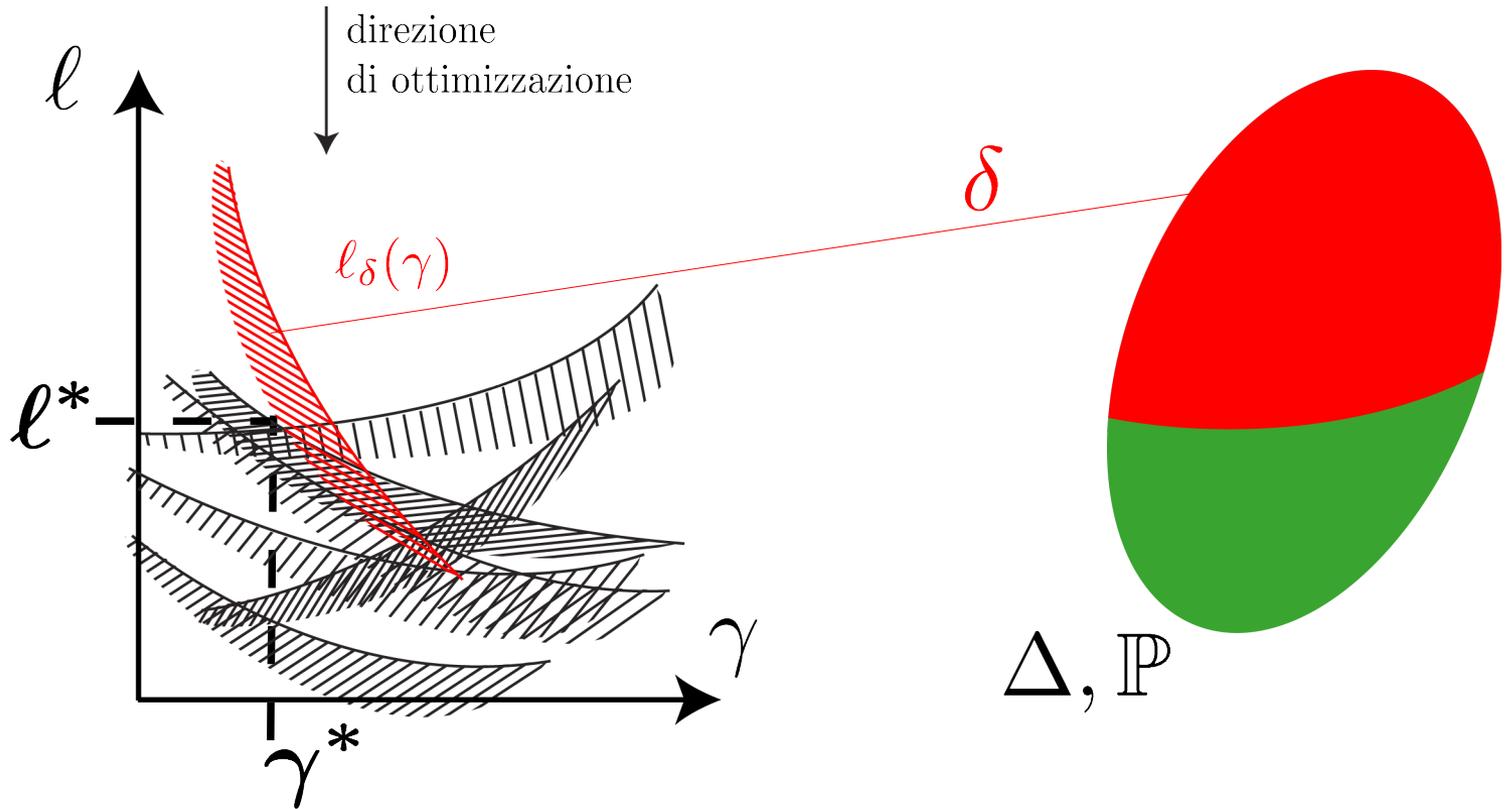
Rischio



Rischio



Rischio



R dipende da γ^* , l^* e \mathbb{P}

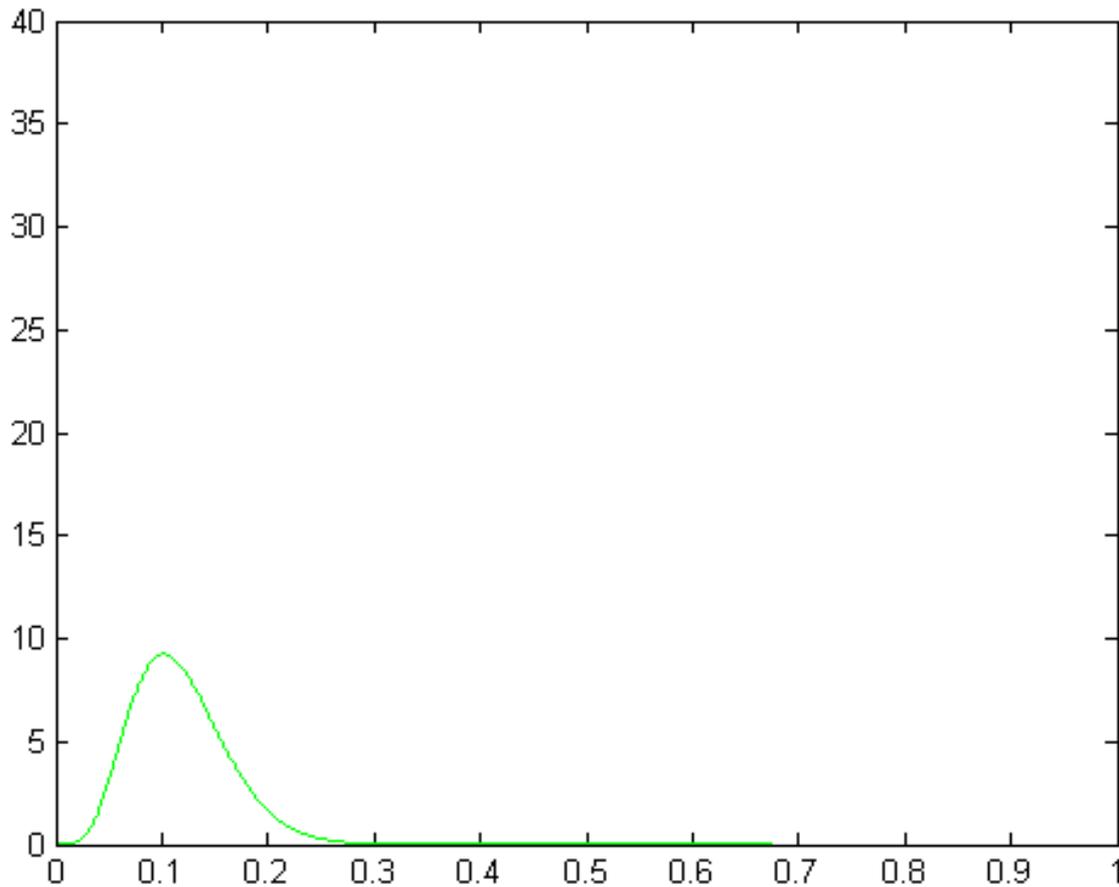
Rischio

$N =$ numero di scenari

$d = \dim(\gamma)$

Rischio

```
betapdf([0:0.01:1], d+1, N-d)
```

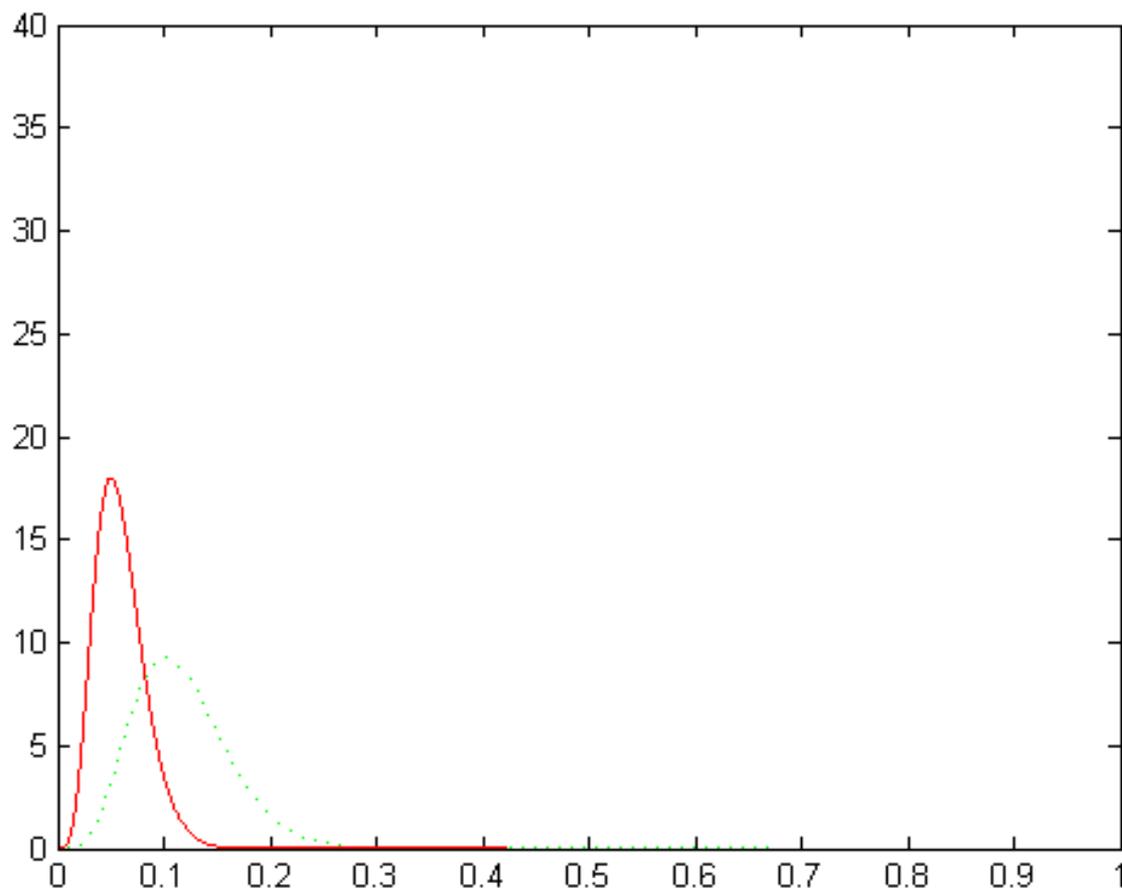


$$d = 5$$

$$N = 50$$

Rischio

```
betapdf([0:0.01:1], d+1, N-d)
```



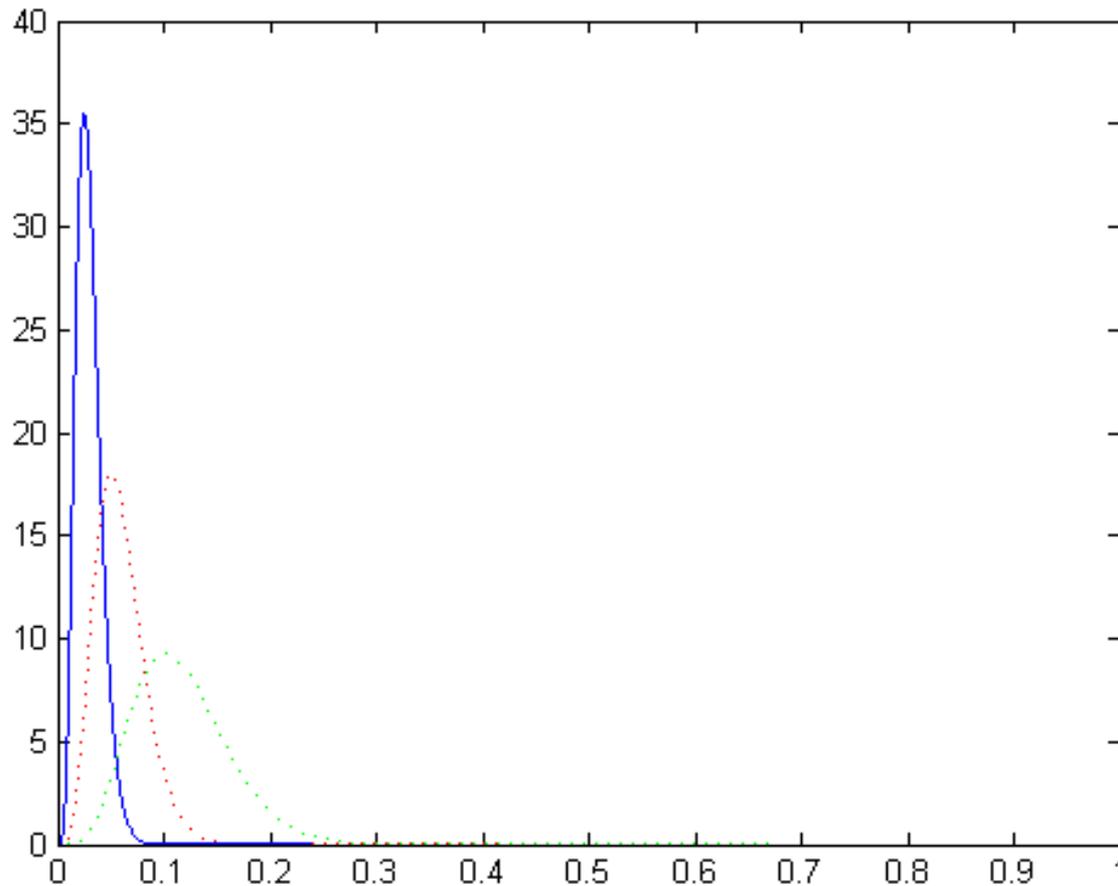
$$d = 5$$

$$N = 100$$

$$N = 50$$

Rischio

```
betapdf([0:0.01:1], d+1, N-d)
```



$$d = 5$$

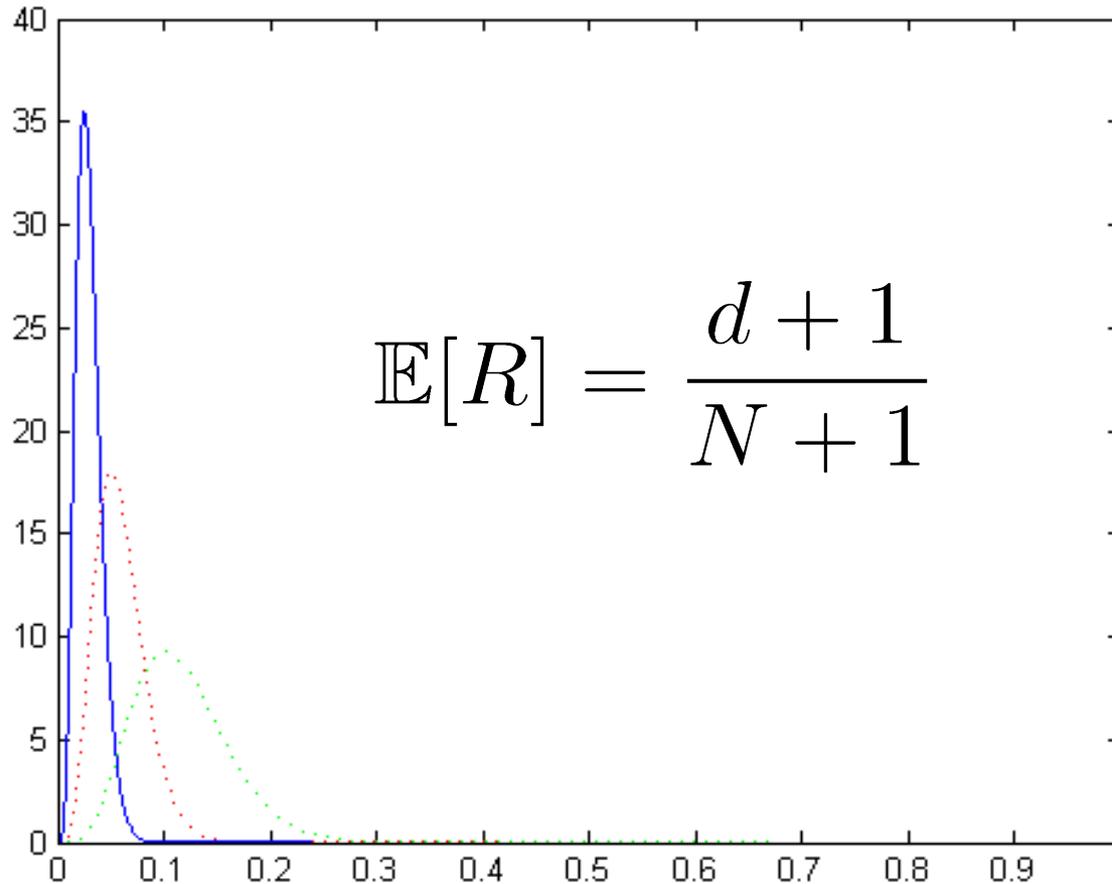
$$N = 200$$

$$N = 100$$

$$N = 50$$

Rischio

```
betapdf([0:0.01:1], d+1, N-d)
```



$$d = 5$$

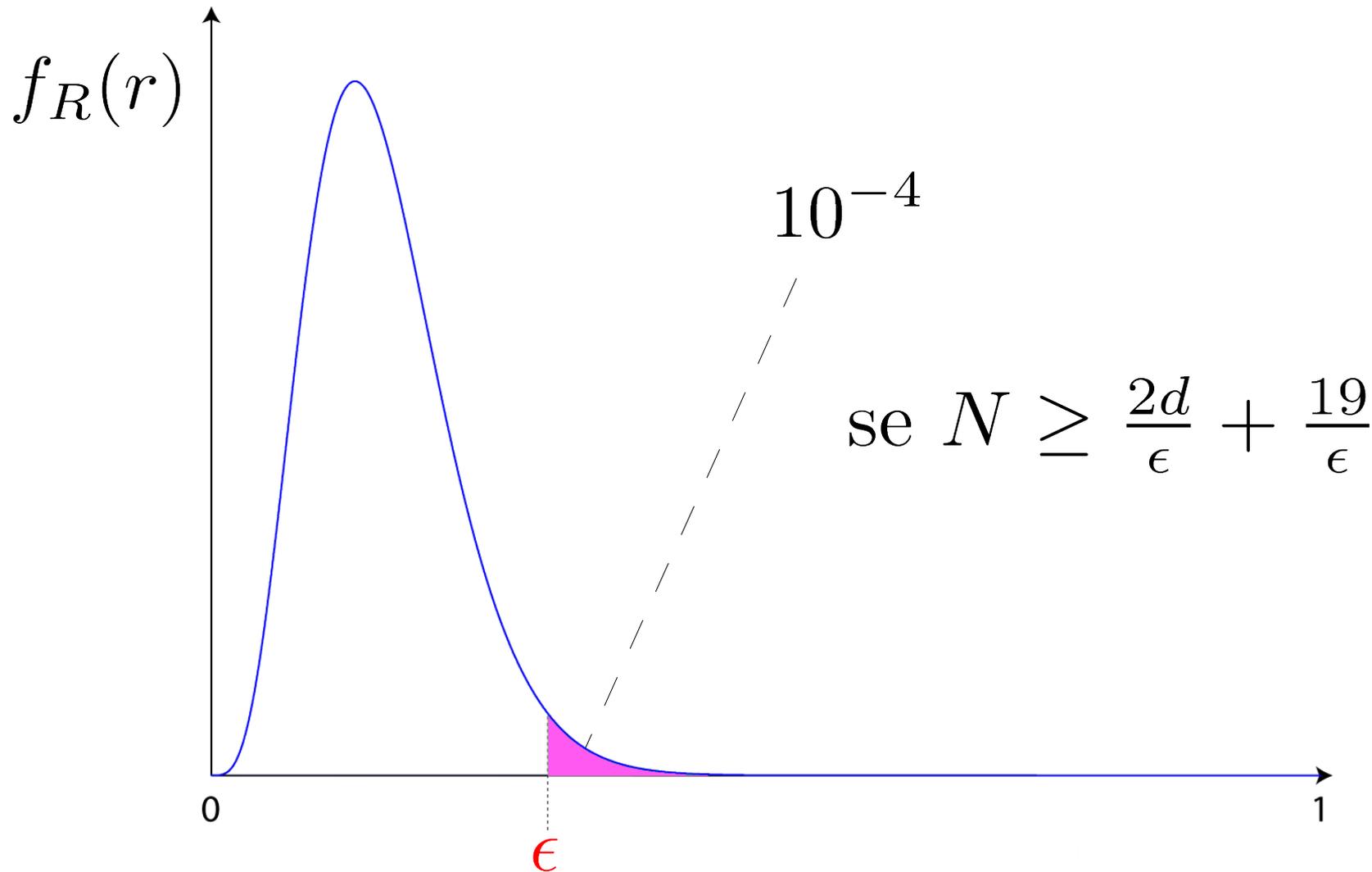
$$\mathbb{E}[R] = \frac{d + 1}{N + 1}$$

$$N = 200$$

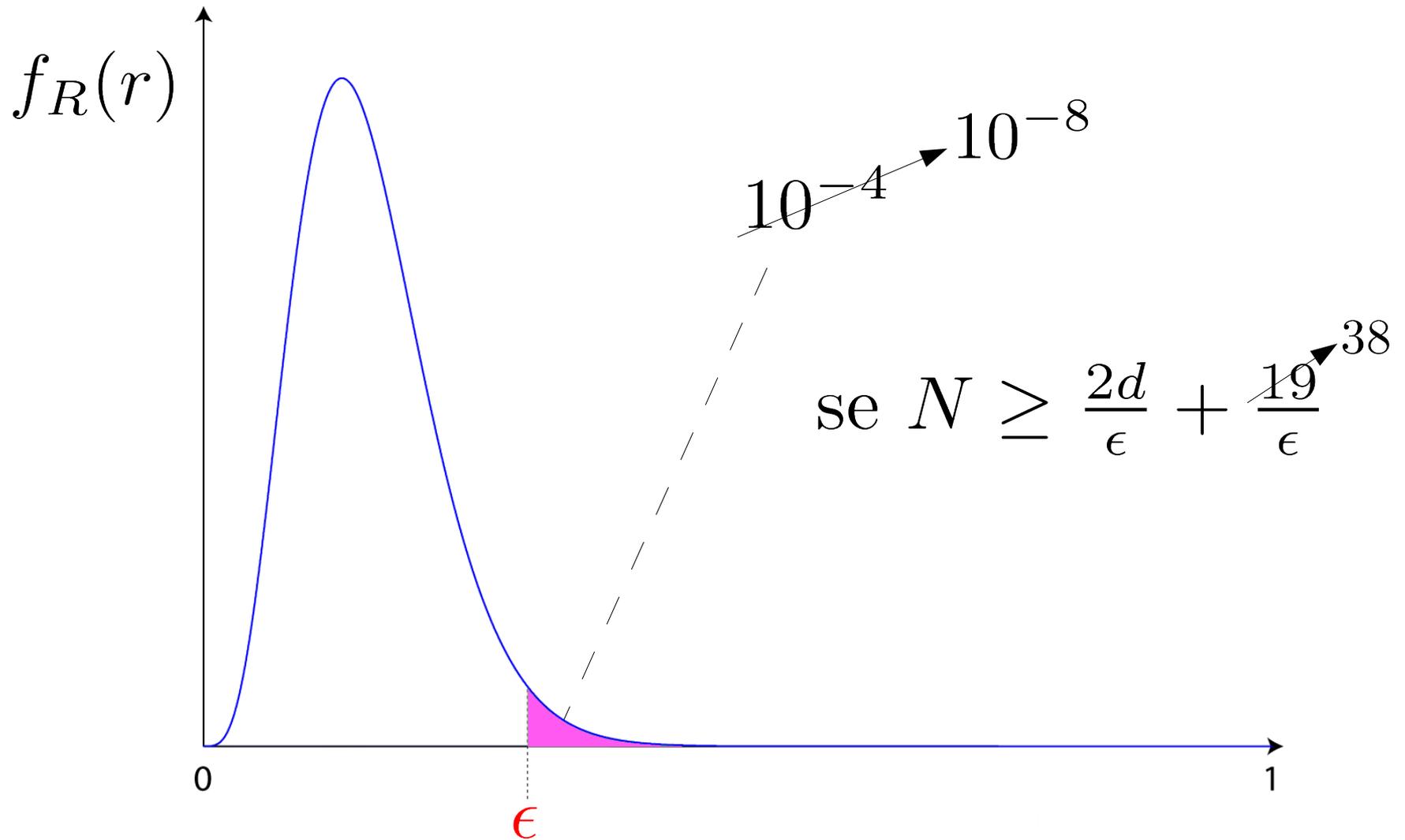
$$N = 100$$

$$N = 50$$

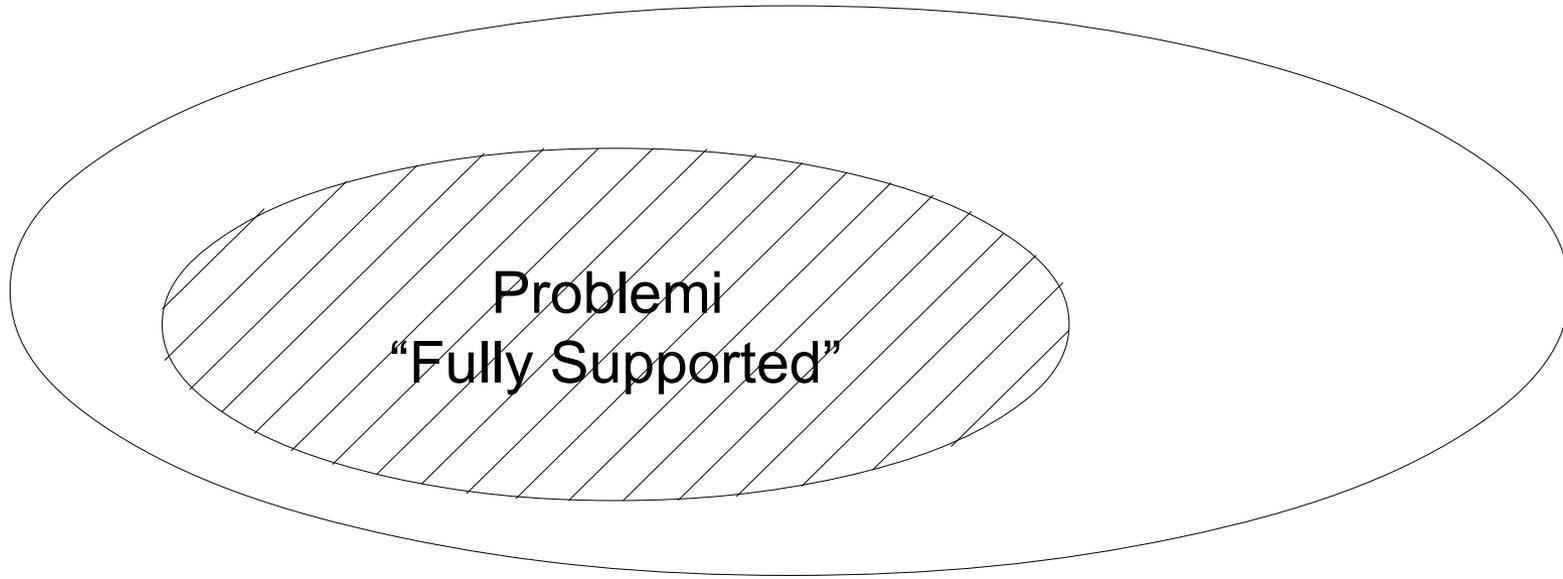
Rischio



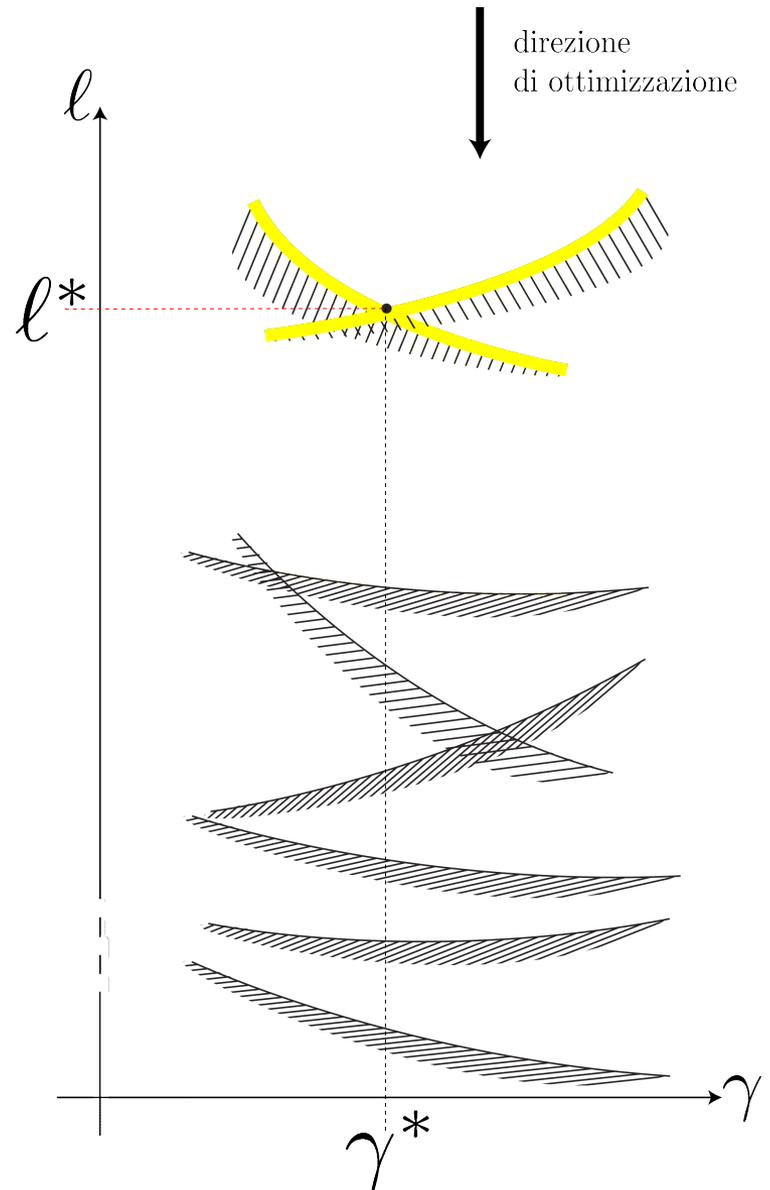
Rischio



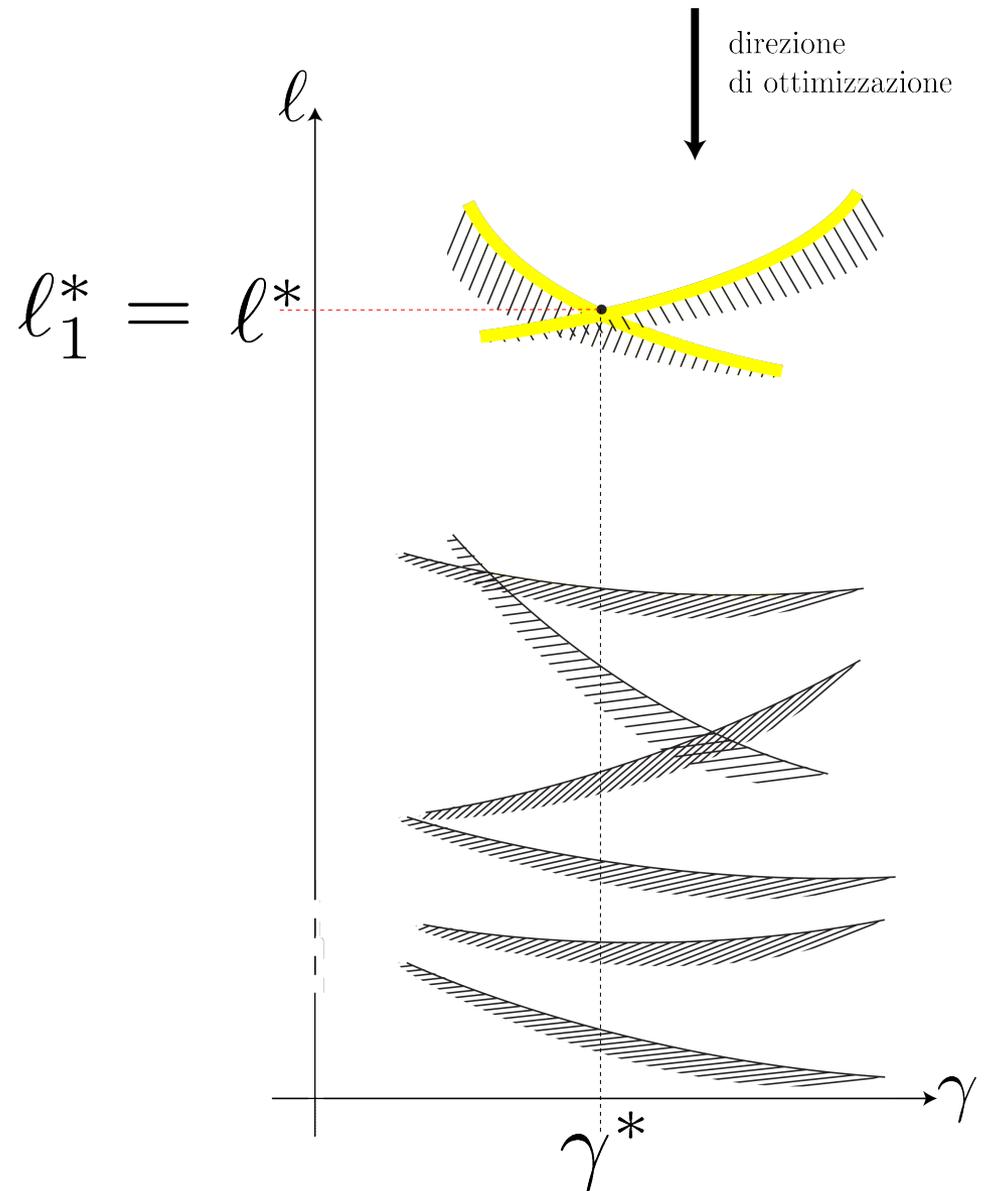
La classe dei problemi min-max



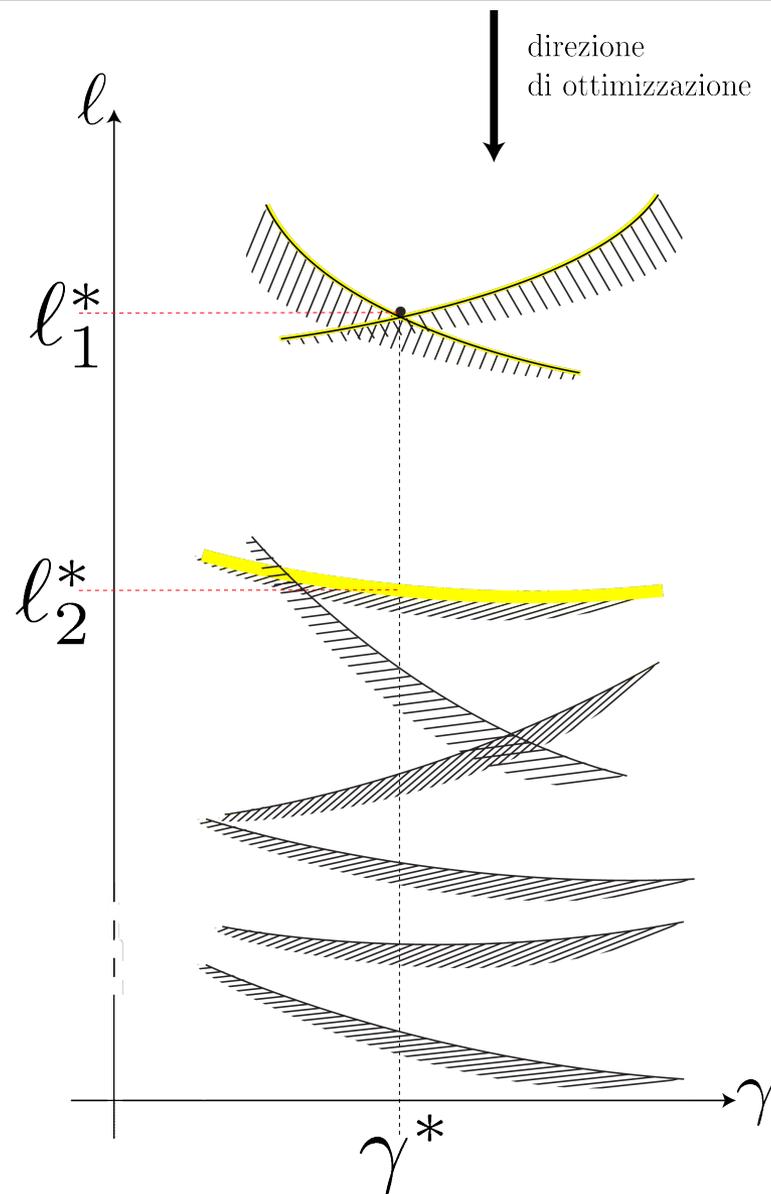
Copertura probabilistica



Copertura probabilistica

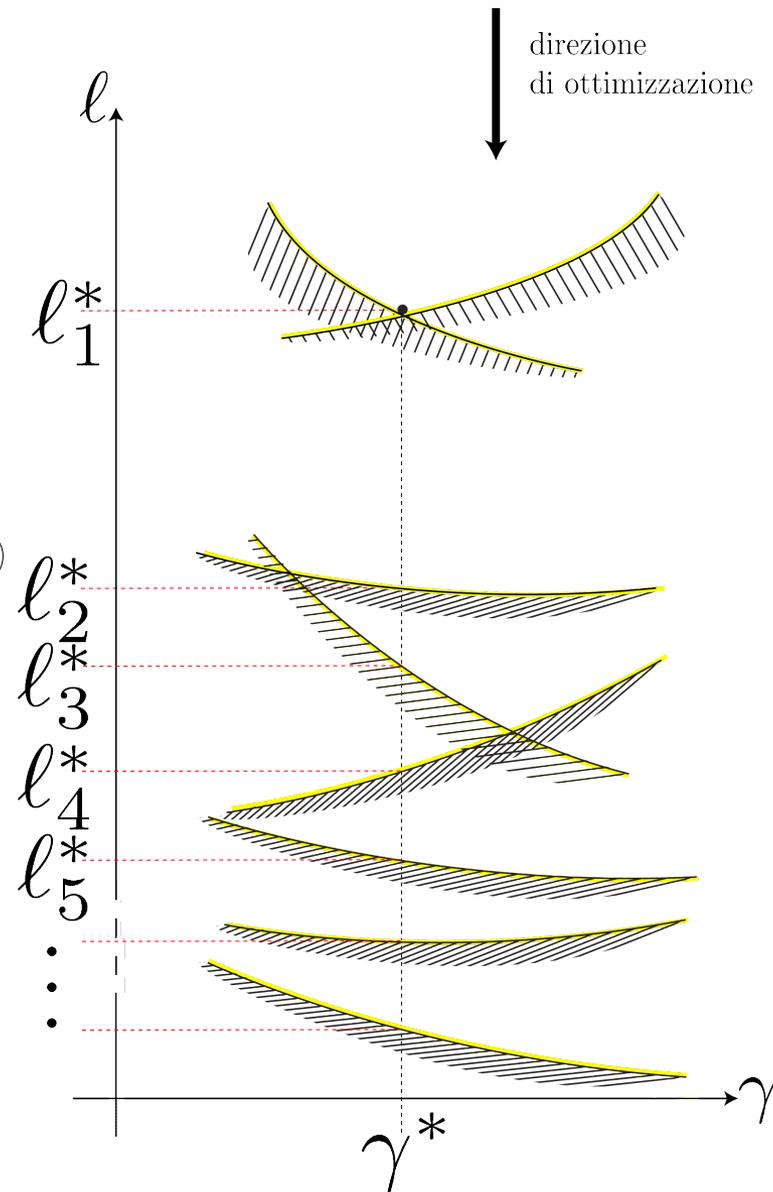


Copertura probabilistica



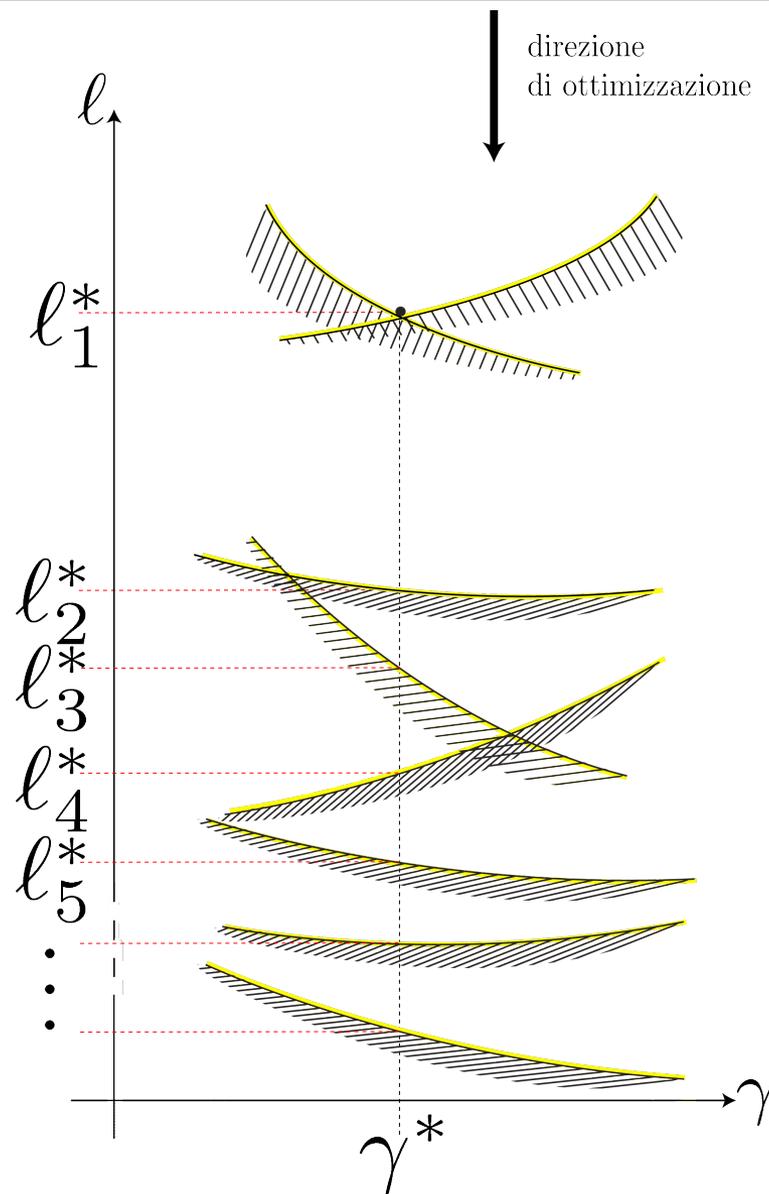
Copertura probabilistica

$l_{\delta(1)}(\gamma^*), l_{\delta(2)}(\gamma^*), \dots, l_{\delta(N)}(\gamma^*)$
 Ordinati!



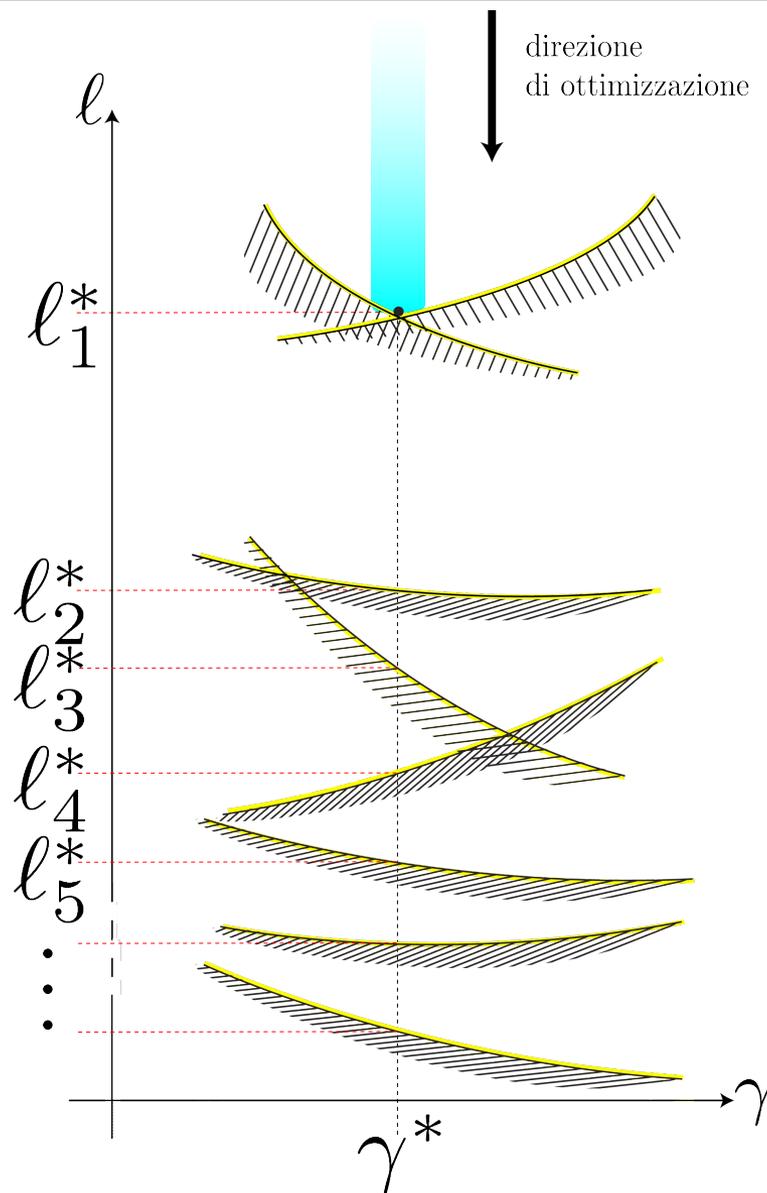
Copertura probabilistica

$$R_k := \mathbb{P}\{\delta : l_\delta(\gamma^*) > l_k^*\}$$



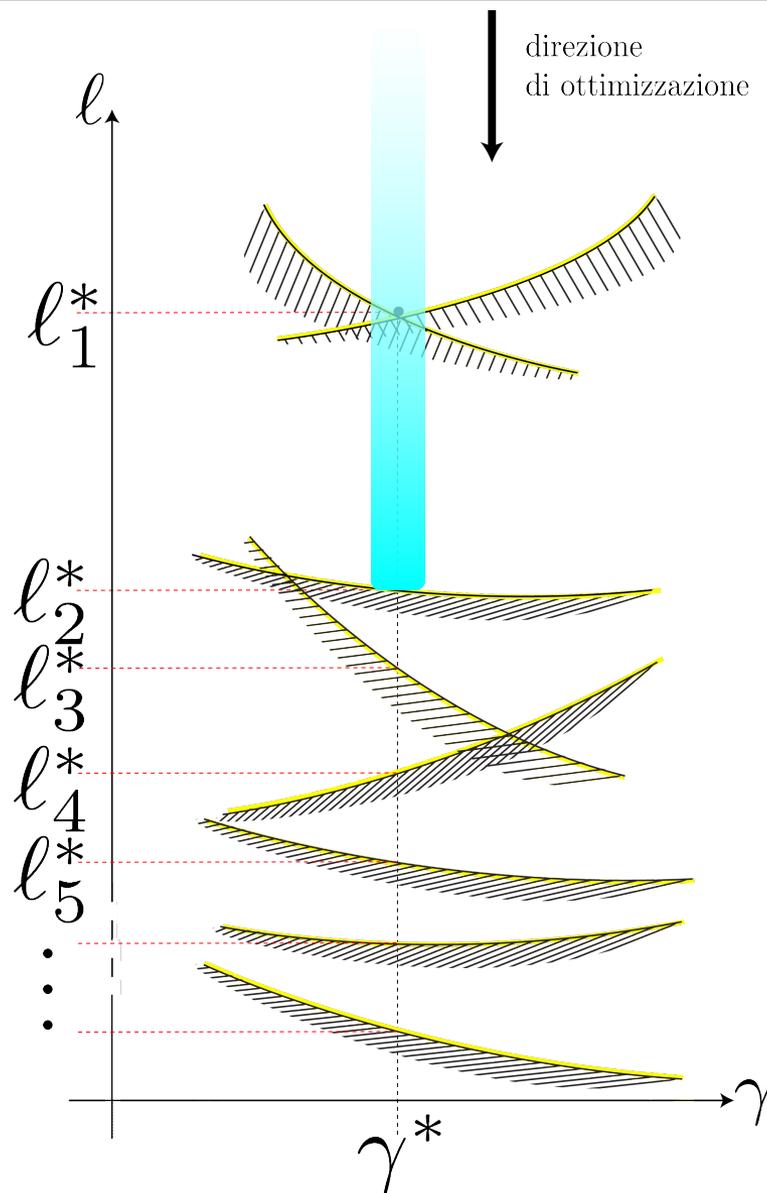
Copertura probabilistica

$$R_k := \mathbb{P}\{\delta : l_\delta(\gamma^*) > l_k^*\}$$

 R_1


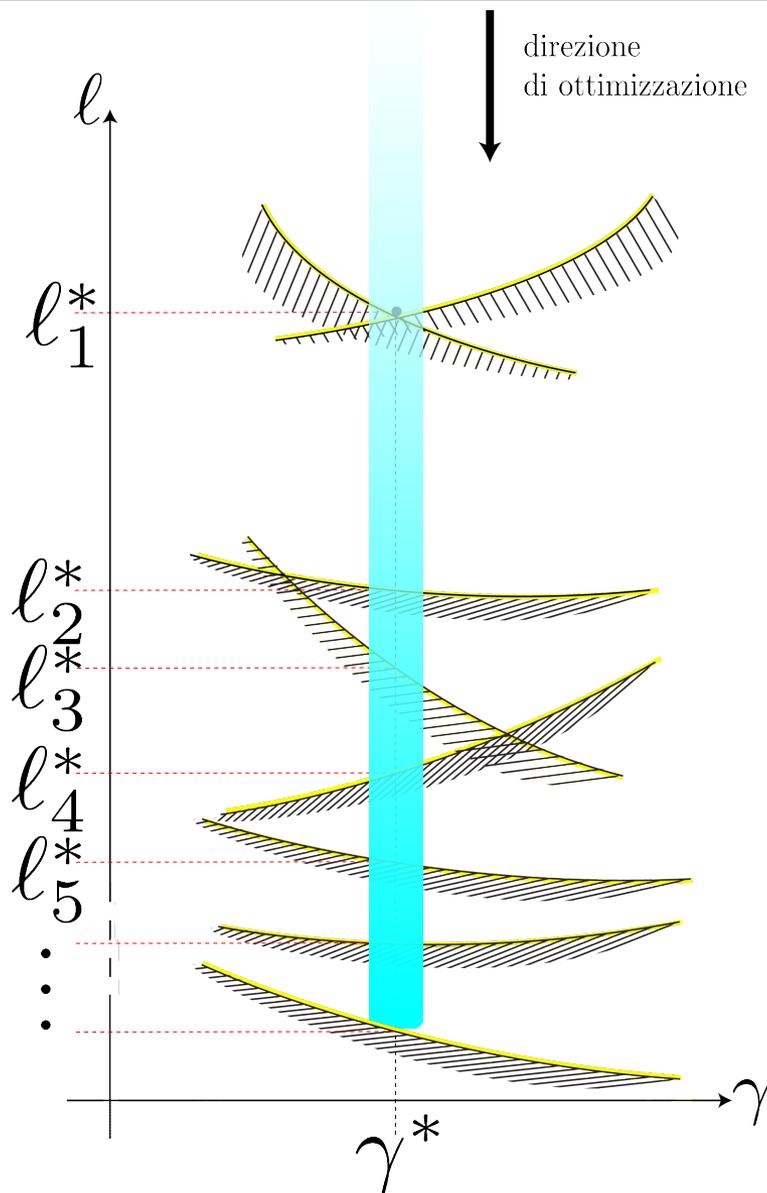
Copertura probabilistica

$$R_k := \mathbb{P}\{\delta : l_\delta(\gamma^*) > l_k^*\}$$

 R_1
 R_2


Copertura probabilistica

$$R_k := \mathbb{P}\{\delta : l_\delta(\gamma^*) > l_k^*\}$$

 R_1
 l_1^*
 R_2
 l_2^*
 R_3
 l_3^*
 R_4
 l_4^*
 R_5
 l_5^*
 \vdots
 \vdots
 \cdot
 \cdot


Distribuzione di Dirichlet ordinata

$$\mathbb{P}^N \{R_1 \leq \epsilon_1, R_2 \leq \epsilon_2, \dots, R_{N-d} \leq \epsilon_{N-d}\}$$

$$= \frac{N!}{d!} \int_0^{\epsilon_1} \int_{r_1}^{\epsilon_2} \cdots \int_{r_{N-d-1}}^{\epsilon_{N-d}} r_1^d dr_{N-d} \cdots dr_2 dr_1$$

Garanzie sui rischi

$$\begin{aligned} \mathbb{P}^N \{R_1 \leq \epsilon_1, R_2 \leq \epsilon_2, \dots, R_{N-d} \leq \epsilon_{N-d}\} \\ = 1 - 10^{-4} \end{aligned}$$

Garanzie sui rischi

$$\begin{aligned} \mathbb{P}^N \{R_1 \leq \epsilon_1, R_2 \leq \epsilon_2, \dots, R_{N-d} \leq \epsilon_{N-d}\} \\ = 1 - 10^{-4} \end{aligned}$$

Garanzie sui rischi

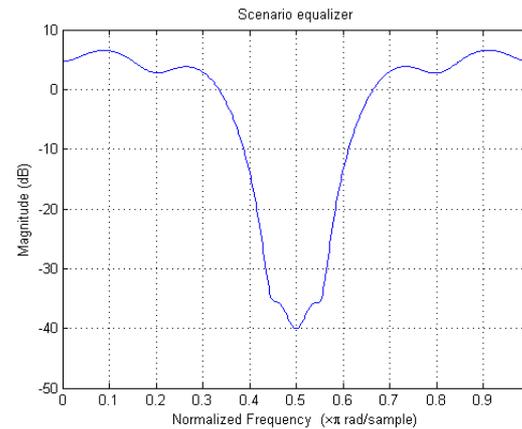
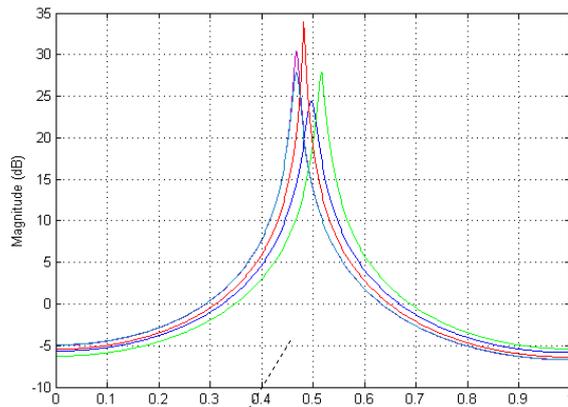
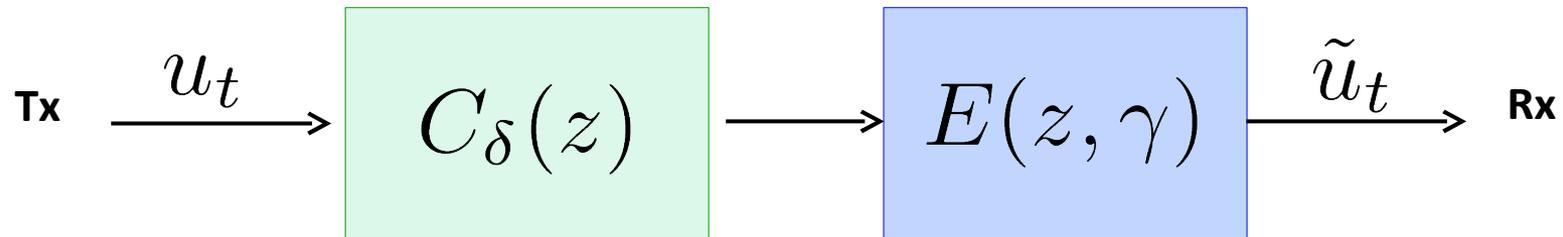
$$\mathbb{P}^N \{R_1 \leq \epsilon_1, R_2 \leq \epsilon_2, \dots, R_{N-d} \leq \epsilon_{N-d}\}$$
$$= 1 - 10^{-4}$$



$$\begin{aligned} R_1 &\leq \epsilon_1 \\ R_2 &\leq \epsilon_2 \\ R_3 &\leq \epsilon_3 \\ &\vdots \end{aligned}$$

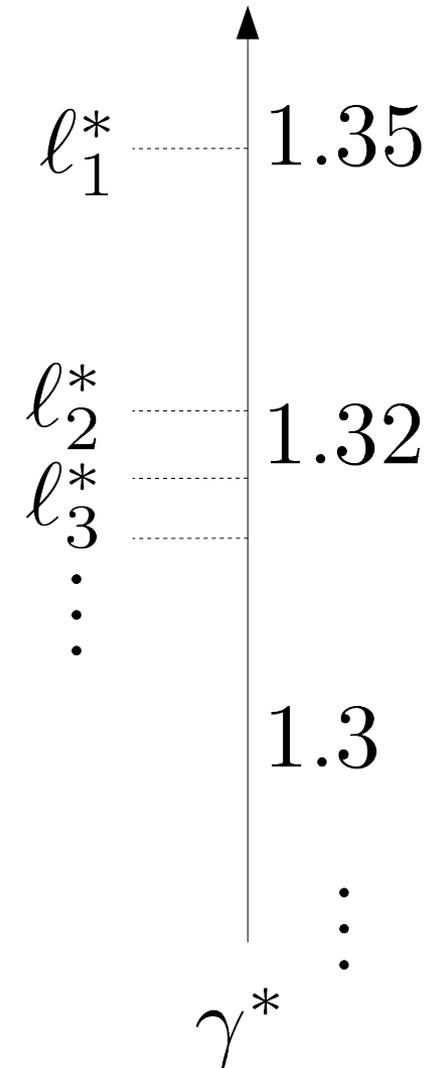
con confidenza
 $1 - 10^{-4}$

Es. - Equalizzazione di canale

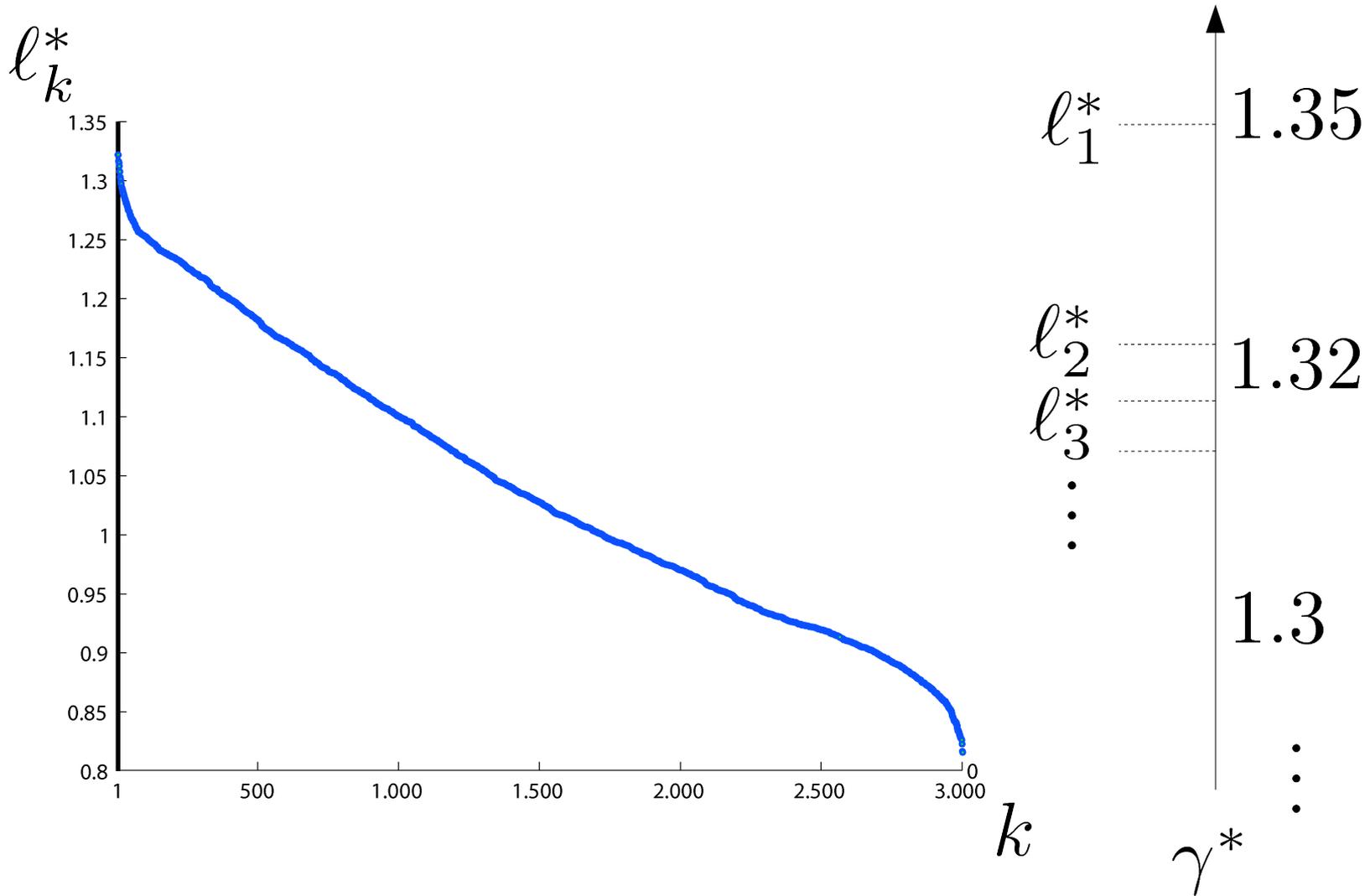

 γ^*

$$l_{\delta(1)}(\gamma^*), l_{\delta(2)}(\gamma^*), \dots, l_{\delta(N)}(\gamma^*)$$

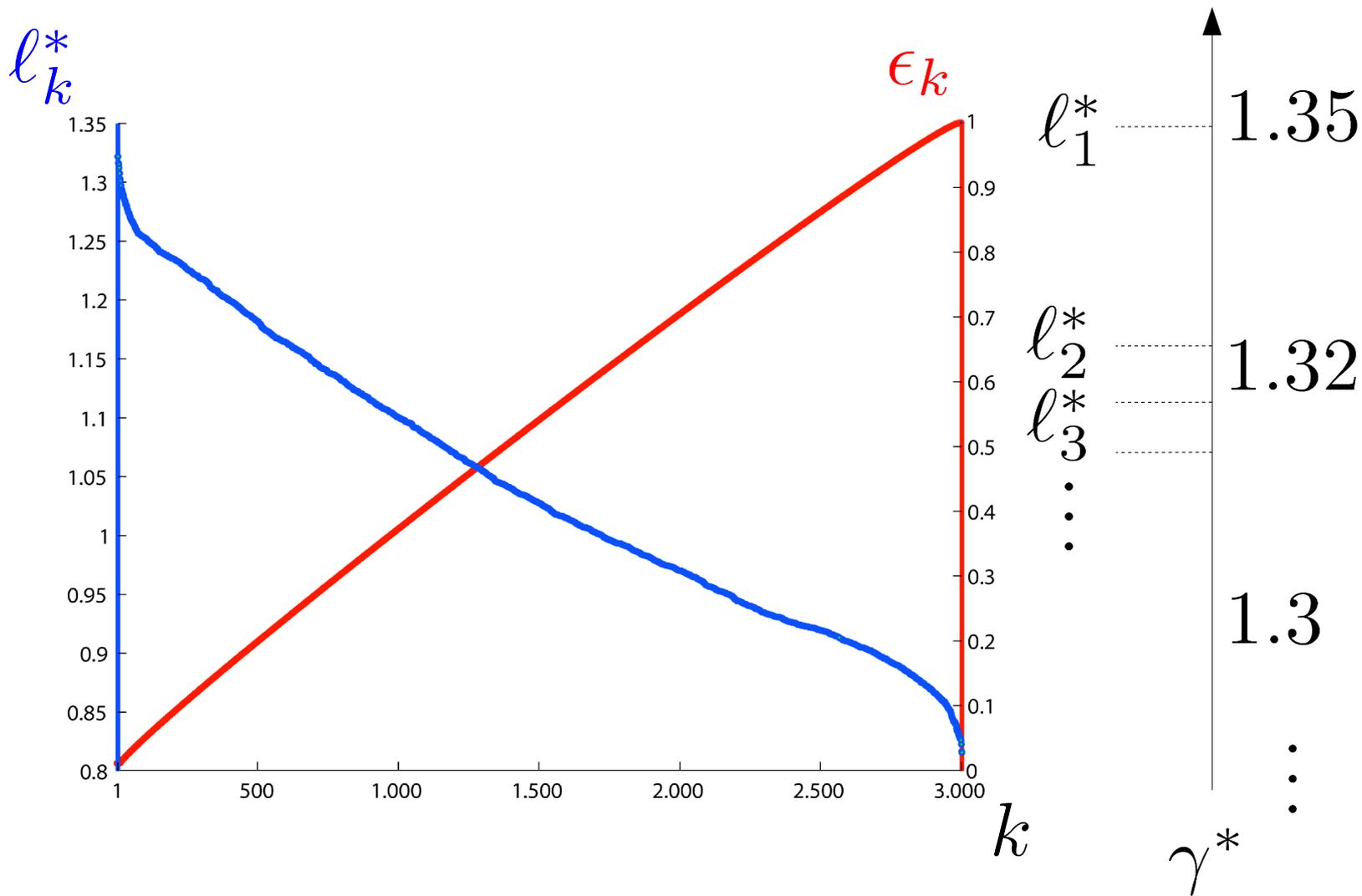
Es. - Equalizzazione di canale



Es. - Equalizzazione di canale



Es. - Equalizzazione di canale

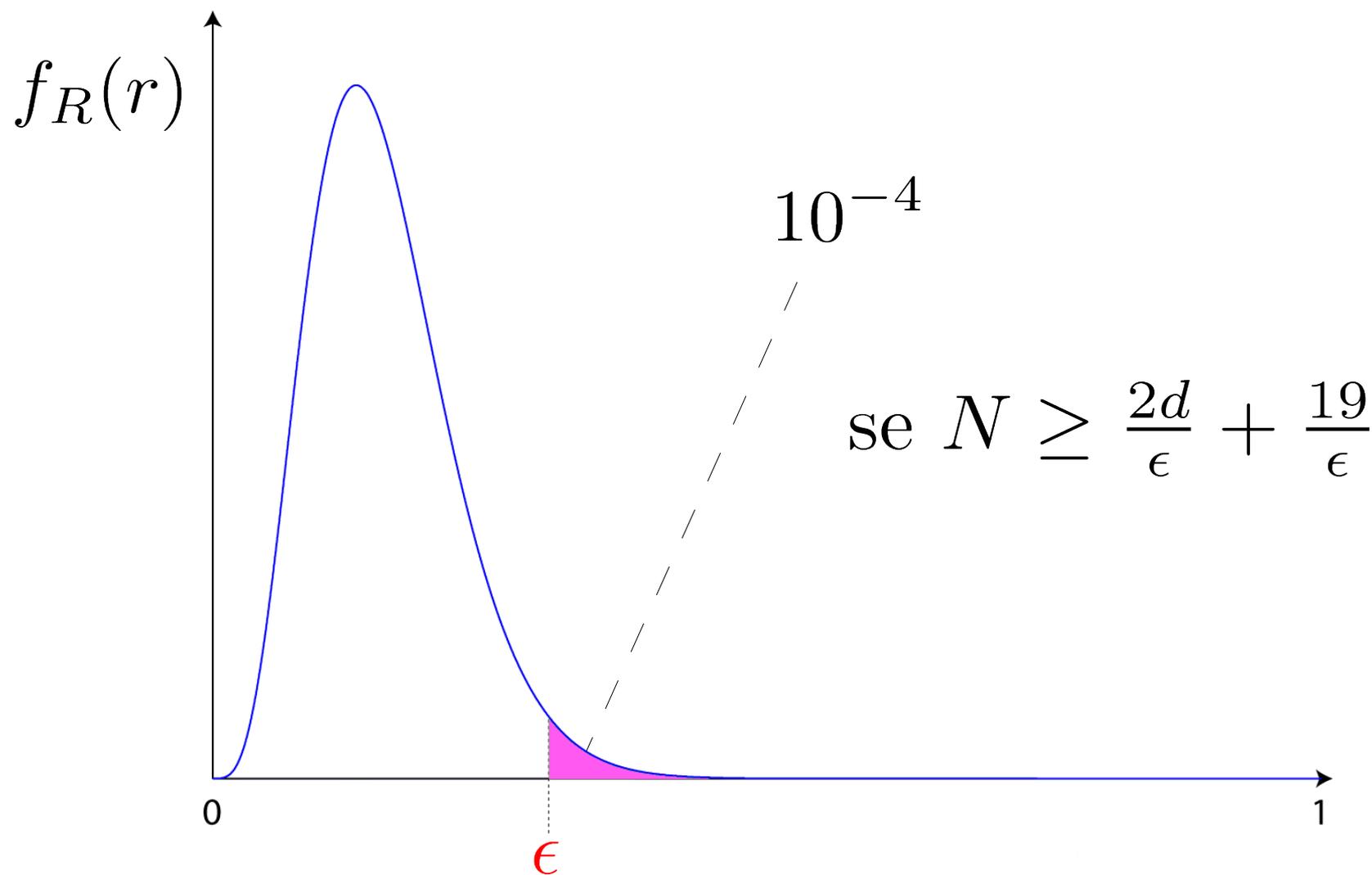


L'approccio a scenario

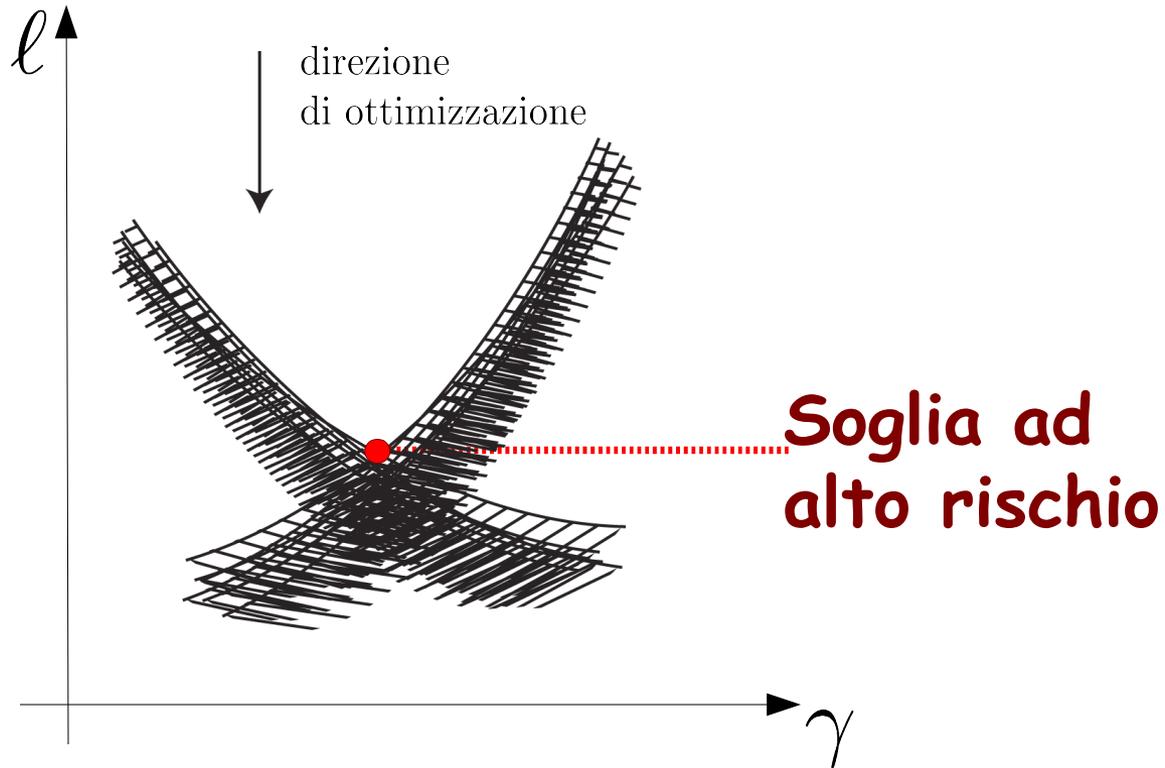
Analisi teorica

ALTRI RISULTATI

Rischio

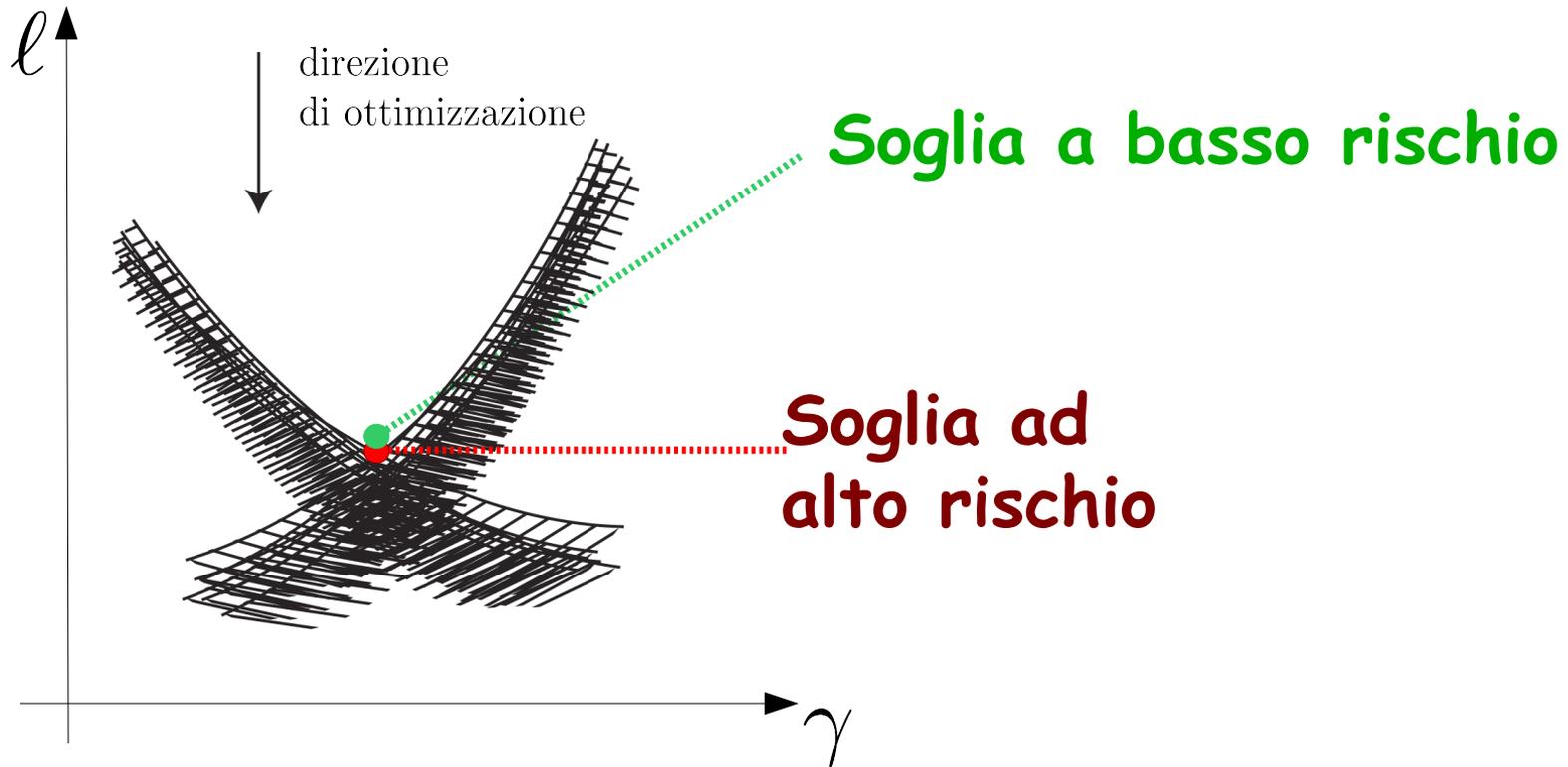


Pochi scenari



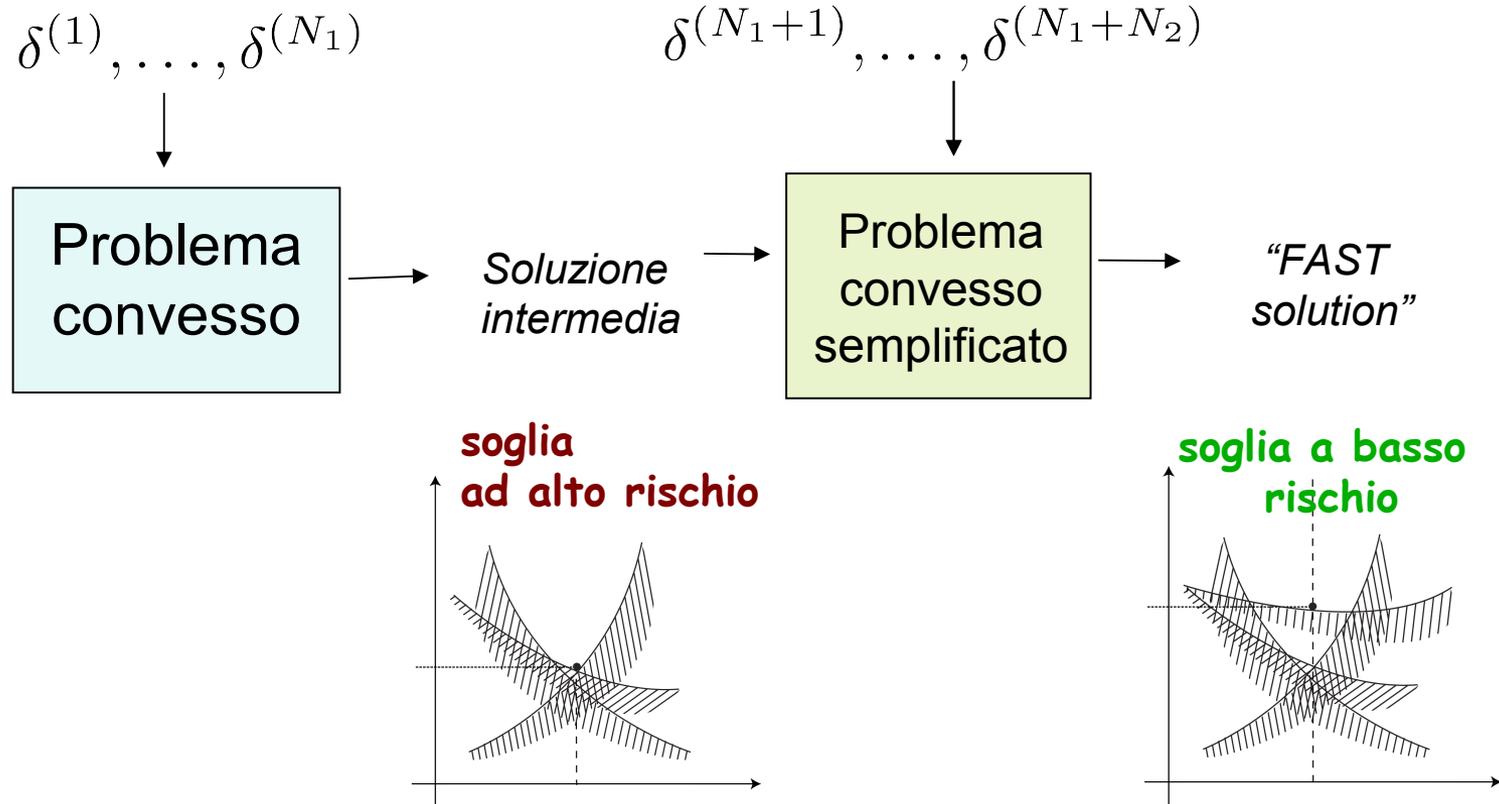
Numero troppo basso di scenari osservati

Eppure capita spesso che...



Numero troppo basso di scenari osservati

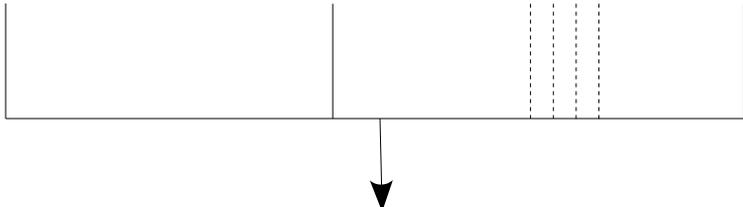
FAST: (Fast Algorithm for the Scenario Technique)



Media sugli scenari

Osservazioni (“scenari”):

$$\delta^{(1)}, \delta^{(2)}, \dots, \delta^{(N)}$$

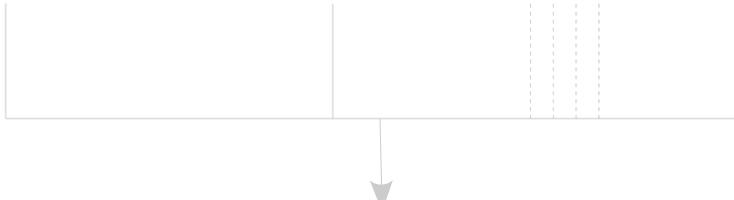
$$l_{\delta^{(1)}}(\gamma), l_{\delta^{(2)}}(\gamma), \dots, l_{\delta^{(N)}}(\gamma)$$


$$\min_{\gamma} \frac{1}{N} \sum_{i=1}^N l_{\delta^{(i)}}(\gamma)$$

Media sugli scenari

$$l_{\delta}(\gamma) = \|A_{\delta}\gamma - b_{\delta}\|^2$$

$$\gamma \in \mathbb{R}^d$$

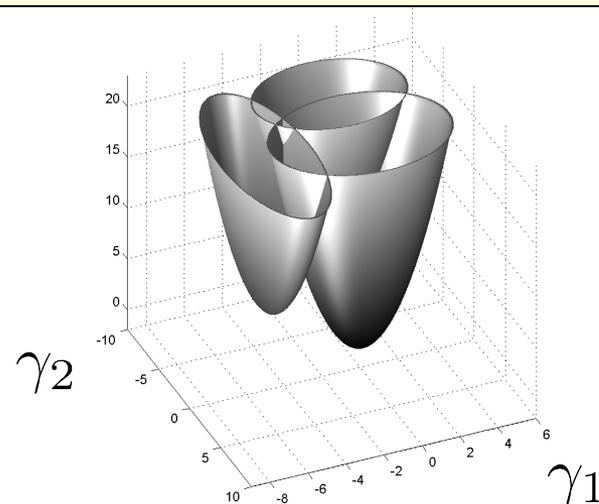
$$l_{\delta^{(1)}}(\gamma), l_{\delta^{(2)}}(\gamma), \dots, l_{\delta^{(N)}}(\gamma)$$


$$\min_{\gamma} \frac{1}{N} \sum_{i=1}^N l_{\delta^{(i)}}(\gamma)$$

Media sugli scenari

$$l_{\delta}(\gamma) = \|A_{\delta}\gamma - b_{\delta}\|^2$$

$$\gamma \in \mathbb{R}^d$$

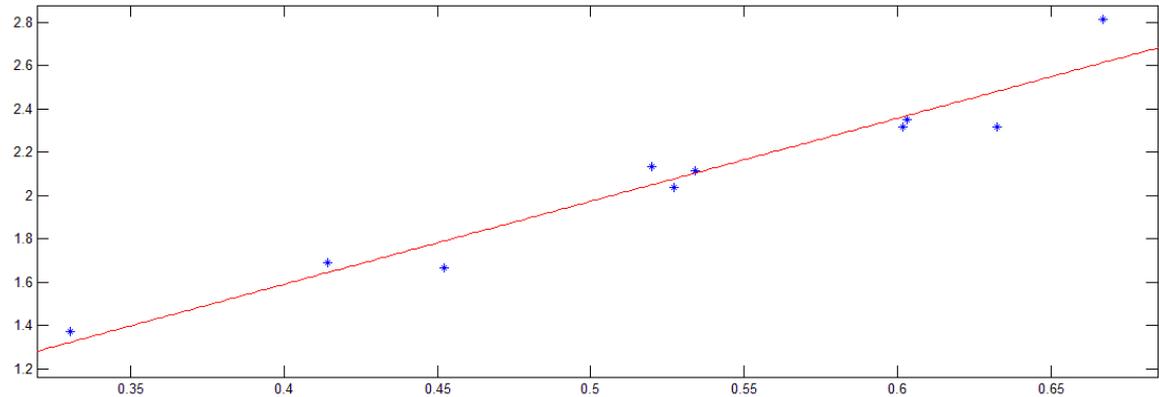


$$l_{\delta^{(1)}}(\gamma), l_{\delta^{(2)}}(\gamma), \dots, l_{\delta^{(N)}}(\gamma)$$

$$\min_{\gamma} \frac{1}{N} \sum_{i=1}^N l_{\delta^{(i)}}(\gamma)$$

Esempi

- Regressione lineare



- Localizzazione di impianti (Weber problem)

- Controllo LQ

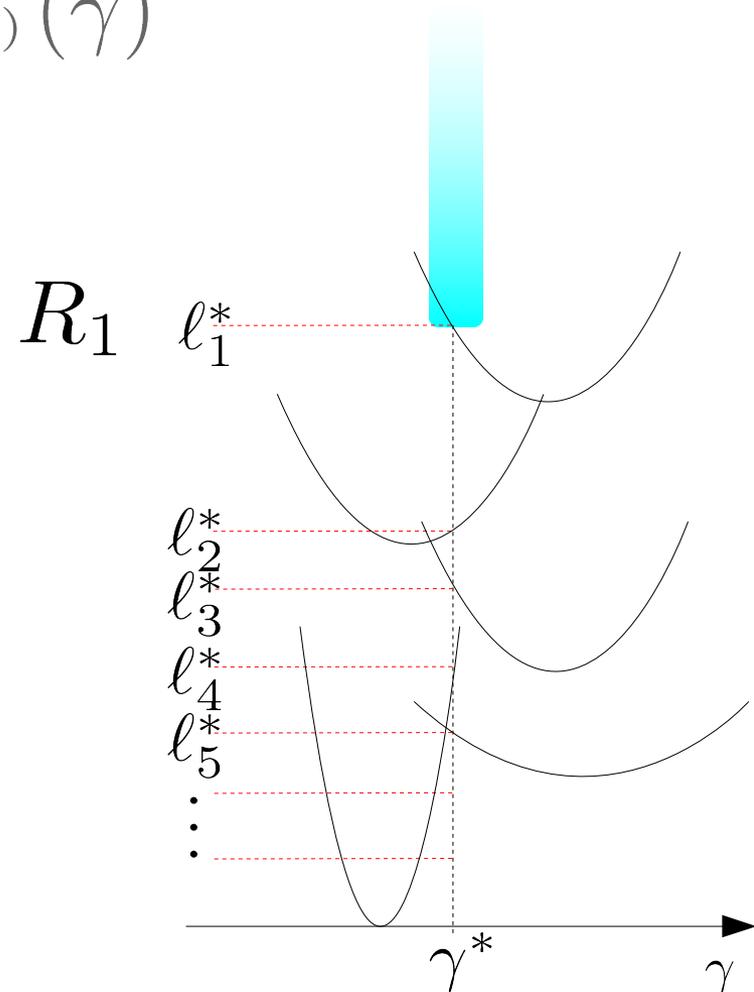
ecc...

Copertura probabilistica

$$\gamma^* = \min_{\gamma} \frac{1}{N} \sum_{i=1, \dots, N} \ell_{\delta(i)}(\gamma)$$

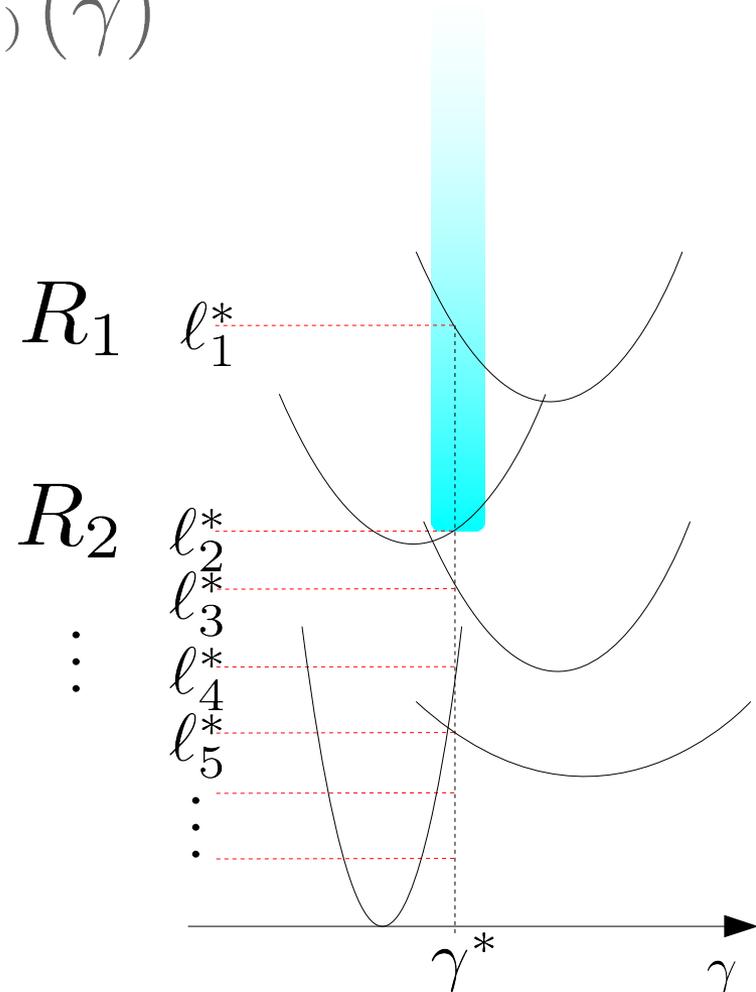
Copertura probabilistica

$$\gamma^* = \min_{\gamma} \frac{1}{N} \sum_{i=1, \dots, N} \ell_{\delta(i)}(\gamma)$$



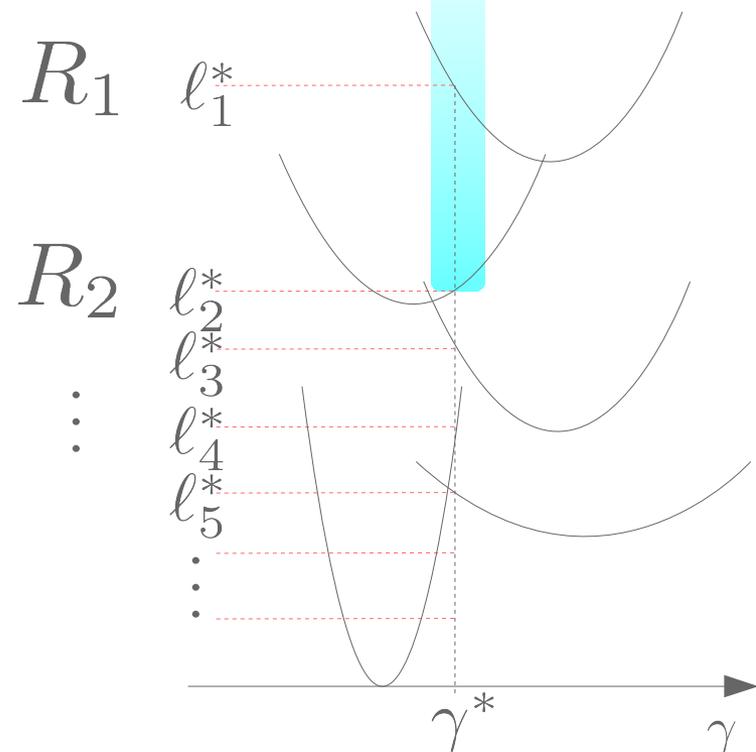
Copertura probabilistica

$$\gamma^* = \min_{\gamma} \frac{1}{N} \sum_{i=1, \dots, N} \ell_{\delta(i)}(\gamma)$$



Copertura probabilistica

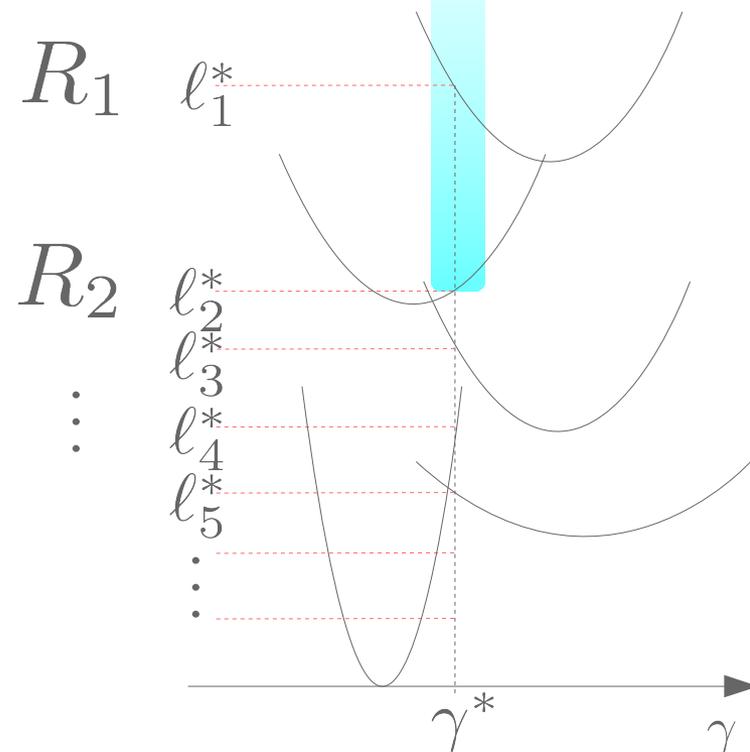
$$\gamma^* = \min_{\gamma} \frac{1}{N} \sum_{i=1, \dots, N} \ell_{\delta(i)}(\gamma)$$



Copertura probabilistica

$$\gamma^* = \min_{\gamma} \frac{1}{N} \sum_{i=1, \dots, N} \ell_{\delta(i)}(\gamma)$$

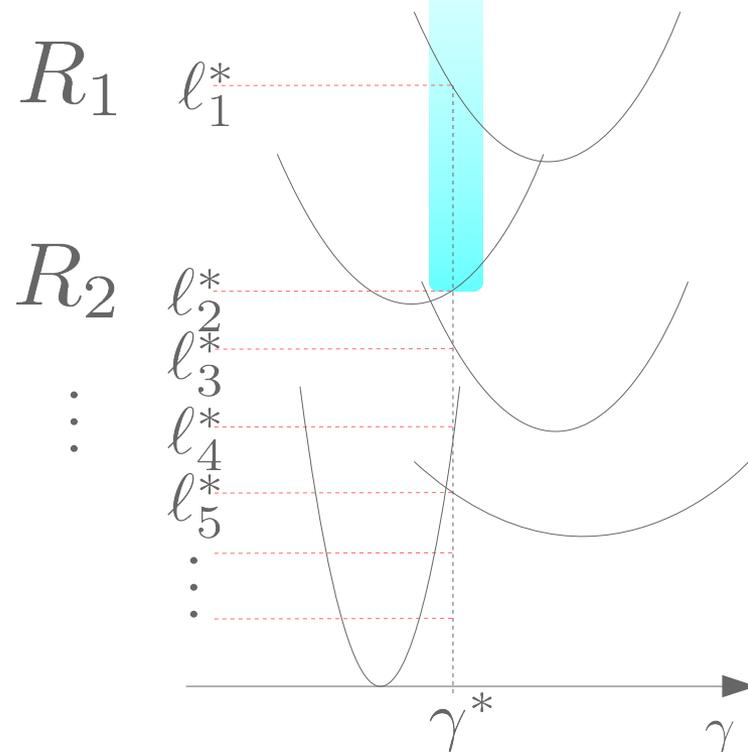
$$\mathbb{E}[R_k] \stackrel{?}{\leq} \frac{k}{N+1}$$



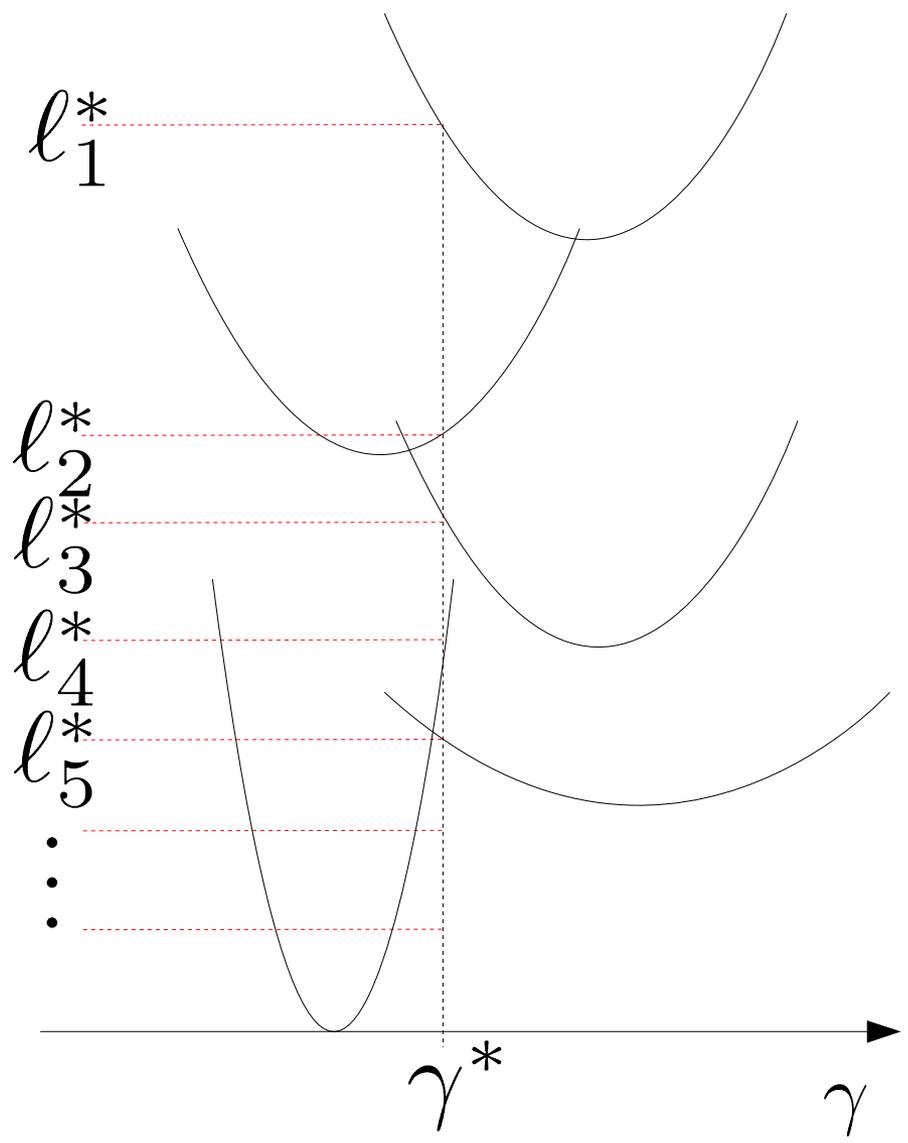
Copertura probabilistica

$$\gamma^* = \min_{\gamma} \frac{1}{N} \sum_{i=1, \dots, N} \ell_{\delta(i)}(\gamma)$$

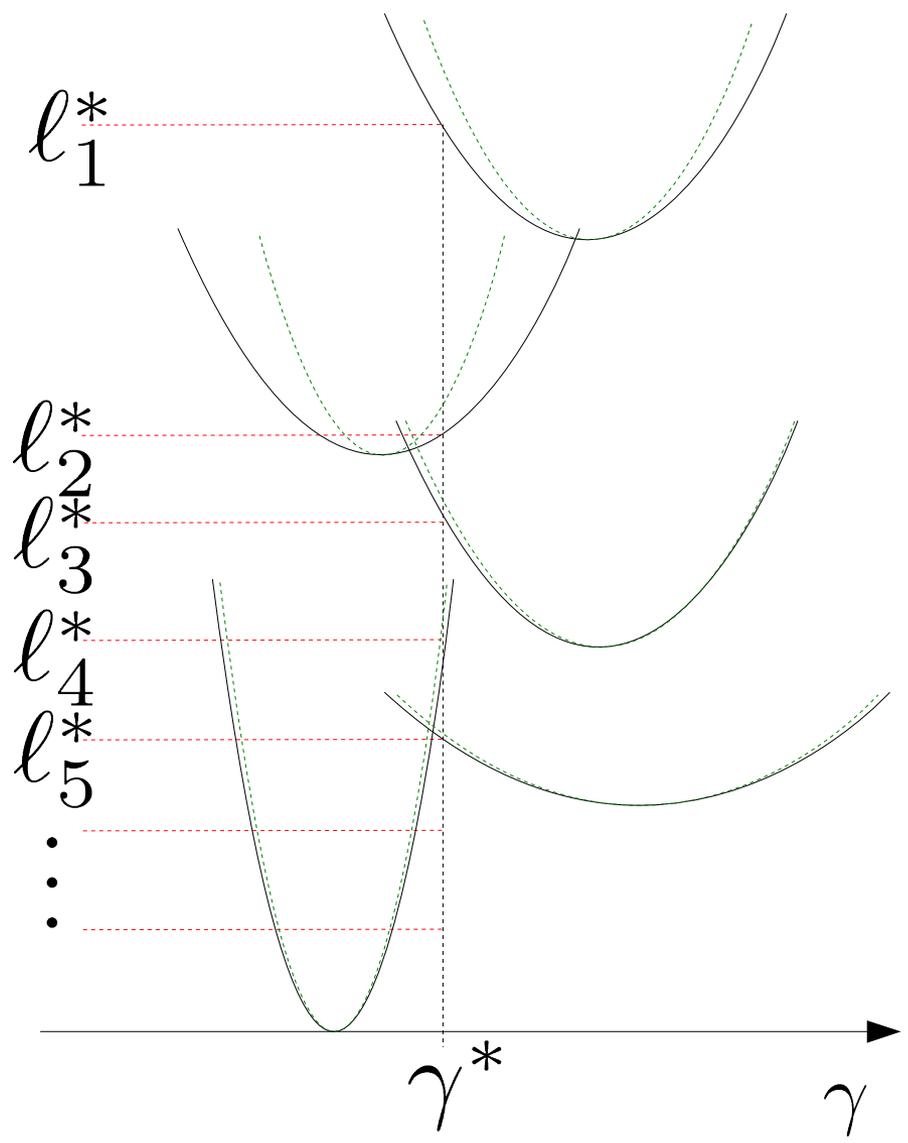
~~$$\mathbb{E}[R_k] \stackrel{?}{\leq} \frac{k}{N+1}$$~~



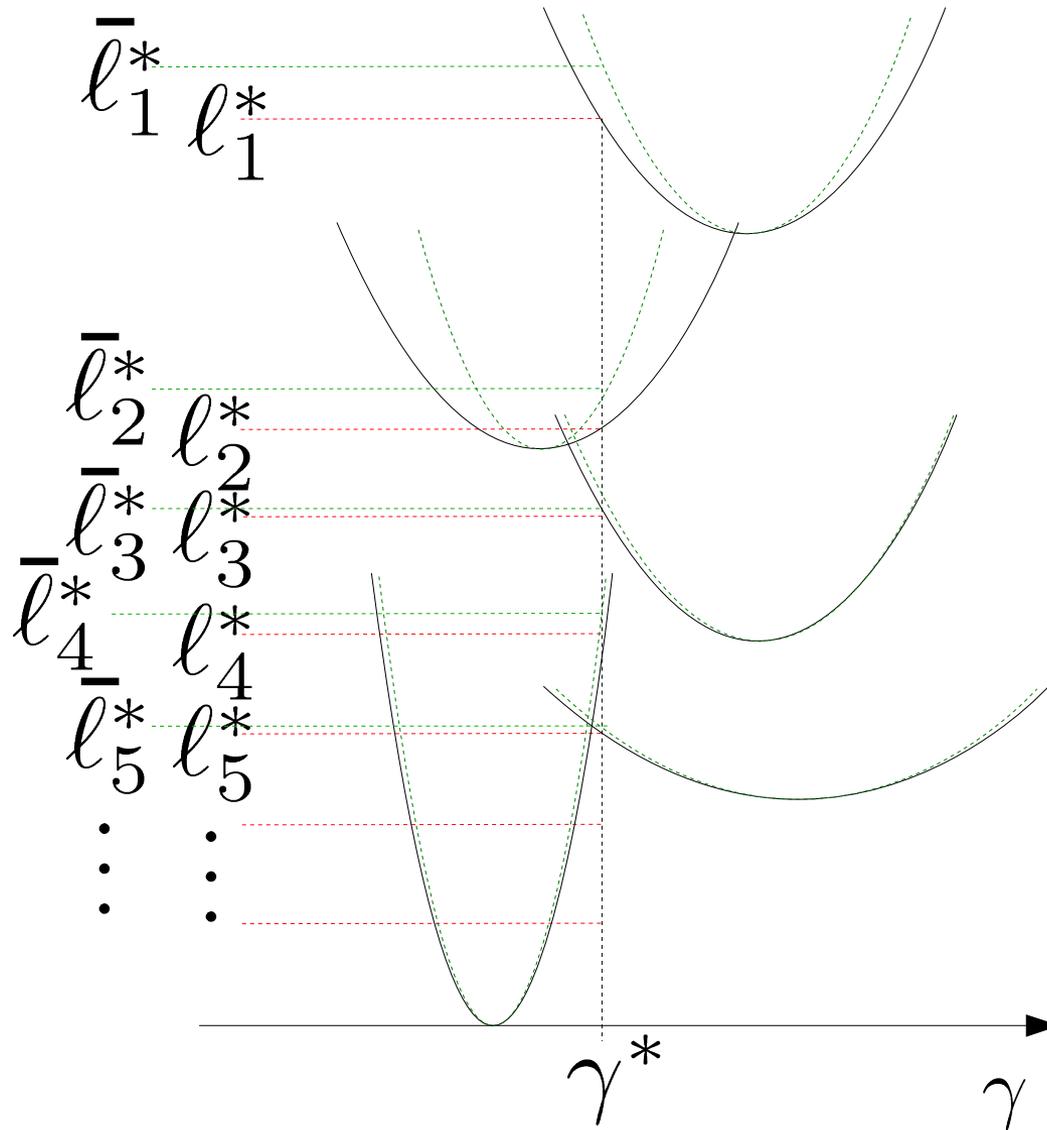
Copertura probabilistica



Copertura probabilistica

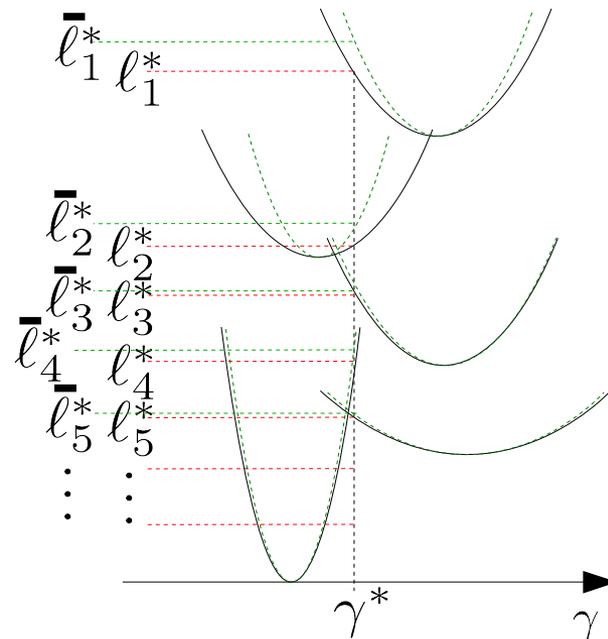


Copertura probabilistica



Copertura probabilistica

$$\mathbb{E}[\bar{R}_k] \leq \frac{k}{N+1} \quad \forall k$$



GRAZIE!

L'approccio a scenario

Analisi teorica

A. Carè, S. Garatti, M. Campi "Randomized min-max optimization: the exact risk of multiple cost levels" 50th IEEE Conference on Decision and Control and European Control Conference, Orlando, Florida, USA, 2011.

A. Carè, S. Garatti, M.C. Campi "FAST: an algorithm for the scenario approach with reduced sample complexity" 18th IFAC World Congress, Milan, Italy, 2011

A. Carè, S. Garatti, M. Campi "Randomized min-max optimization: the exact risk of multiple cost levels" 50th IEEE Conference on Decision and Control and European Control Conference, Orlando, Florida, USA, 2011.

A. Carè, S. Garatti, M.C. Campi "FAST: an algorithm for the scenario approach with reduced sample complexity" 18th IFAC World Congress, Milan, Italy, 2011

A. Carè, S. Garatti, M.C. Campi "Least Squares estimates and the coverage of Least Squares costs" sottomesso – 52nd IEEE CDC, 2013

A. Carè, S. Garatti, M. Campi "Randomized min-max optimization: the exact risk of multiple cost levels" 50th IEEE Conference on Decision and Control and European Control Conference, Orlando, Florida, USA, 2011.

A. Carè, S. Garatti, M.C. Campi "FAST: an algorithm for the scenario approach with reduced sample complexity" 18th IFAC World Congress, Milan, Italy, 2011

A. Carè, S. Garatti, M.C. Campi "Least Squares estimates and the coverage of Least Squares costs" sottomesso – 52nd IEEE CDC, 2013

Versioni estese

A. Carè, S. Garatti, M.C. Campi "FAST – Fast Algorithm for the Scenario Technique" sottomesso – Operations Research 2013

A. Carè, S. Garatti, M.C. Campi
"On the reliability of sampled-based min-max optimization"
in revisione interna – 2012

A. Carè, S. Garatti, M.C. Campi "Least Squares estimates and the coverage of Least Squares costs" bozza – 2013

M. Baronio, A. Sorgato, G. Perone, A. Candiani, A. Carè, M. Campi, F. Baronio
"Waveform combination measures of ventricular fibrillation to predict defibrillation"
in revisione interna – Università di Brescia 2010