



# The Number and the Infinite

(a short introduction to Georg Cantor's theory)

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# Georg Cantor



*Georg Cantor*

**(Saint Petersburg 1845 – Halle on the Saale river 1918)**

What is a Number?



WIKIPEDIA  
*The Free Encyclopedia*

<<A number is a mathematical object  
used in **counting** [...]>>



WIKIPEDIA  
*The Free Encyclopedia*

<<A number is a mathematical object  
used in **counting** [...]>>

***counting?***

- Number as "sum" (aggregated)

- Number as "sum" (aggregated)

- Number as unity

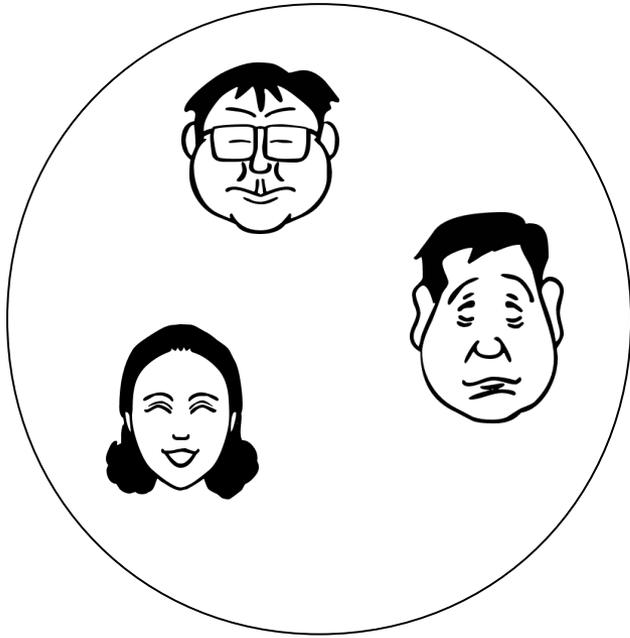
Number as unity

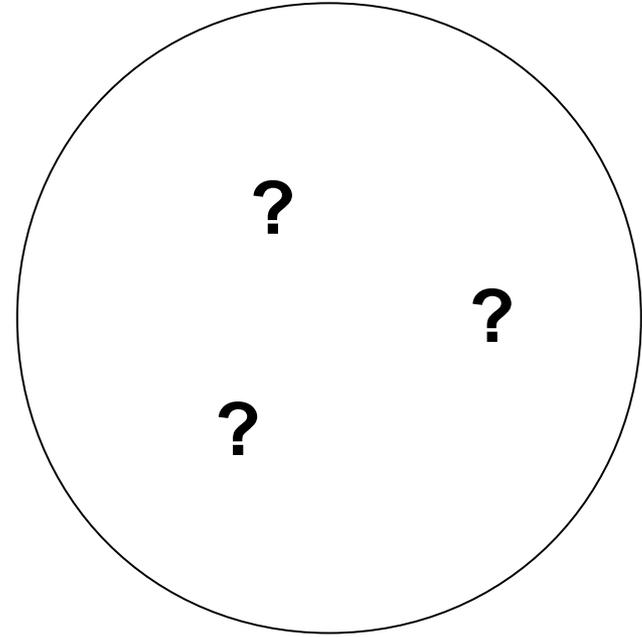
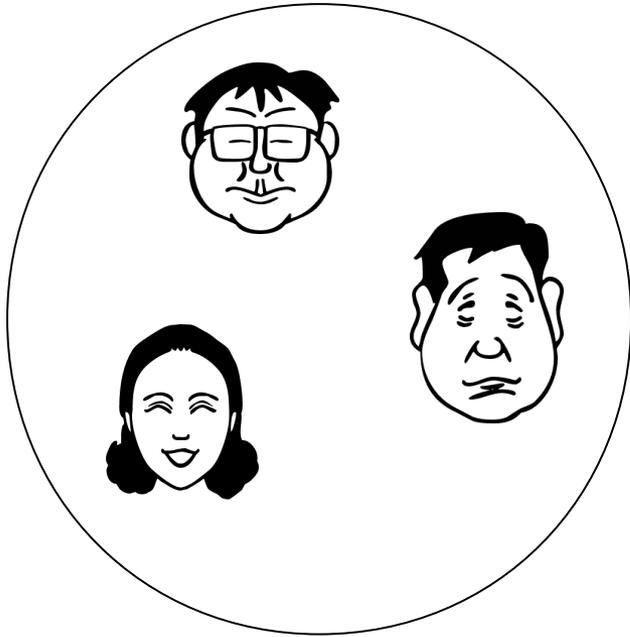
- Αριθμός arithmos
- All is number

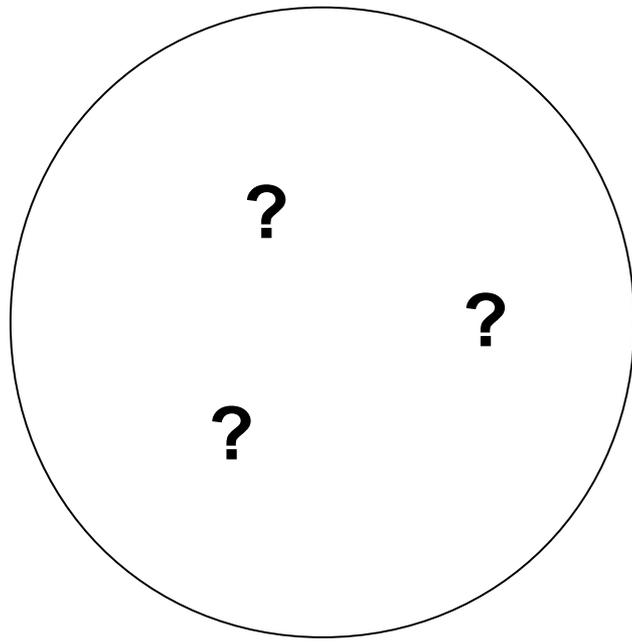
*Pythagorean position*

Number as "sum" (aggregated)

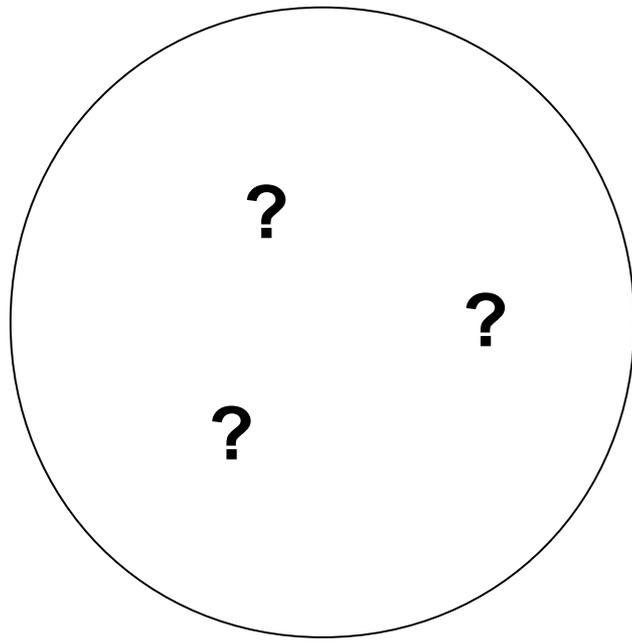
- Newton, Euclid,...





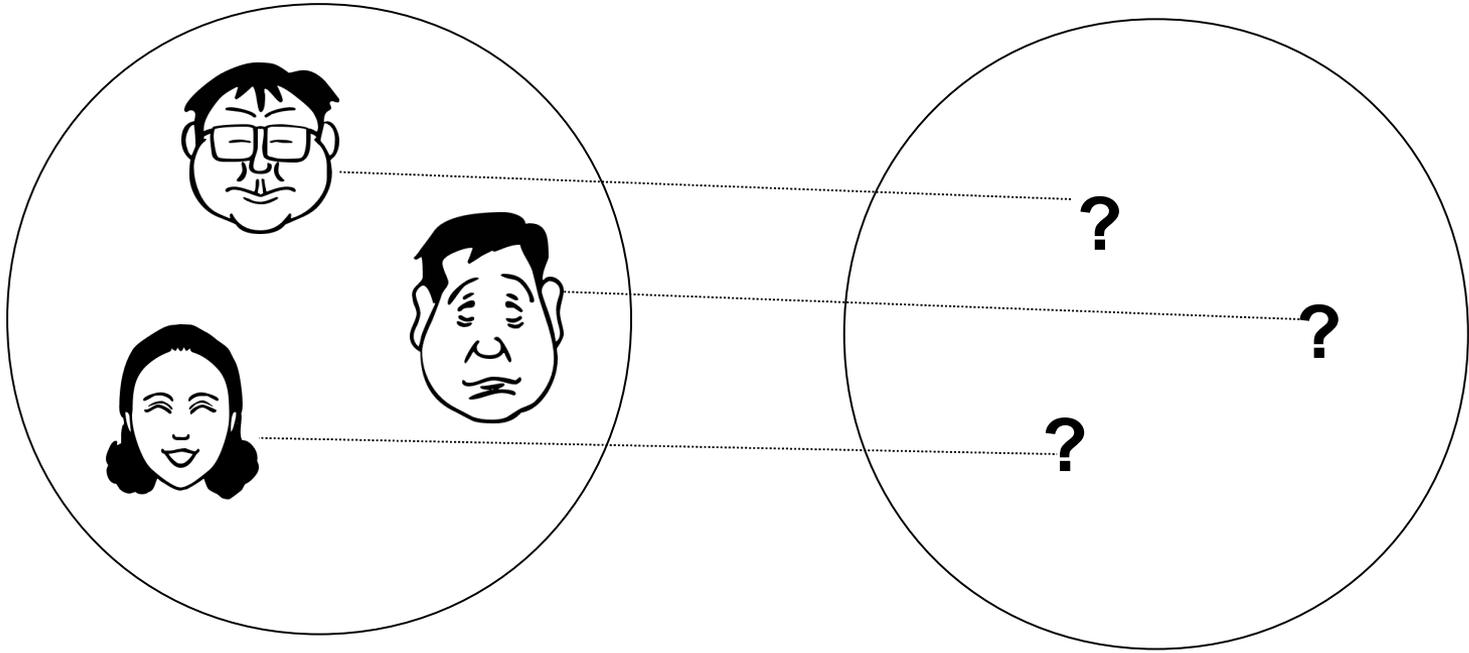


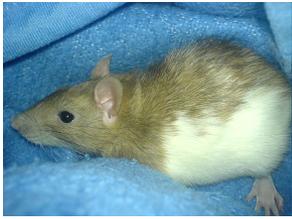
CARDINAL NUMBER



CARDINAL NUMBER

**3**





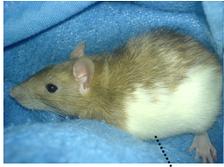


IV



IV





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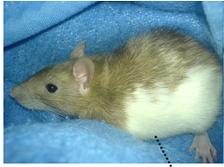
$\leq$



$\leq$



**?  $\leq$  ?  $\leq$  ?**



≧



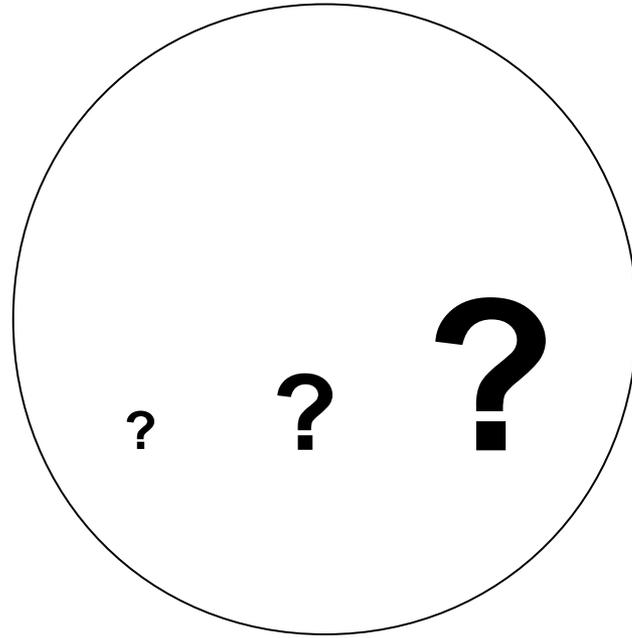
≧



?

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?



ORDINAL NUMBER

Let's play a game

# Let's play a game

Tell me a number:  $N=...$

# Let's play a game

Tell me a number:  $N=...$

Now tell me a number greater than  $N+1$ .

# Let's play a game

Tell me a number:  $N=...$

Now tell me a number greater than  $N+1$ .

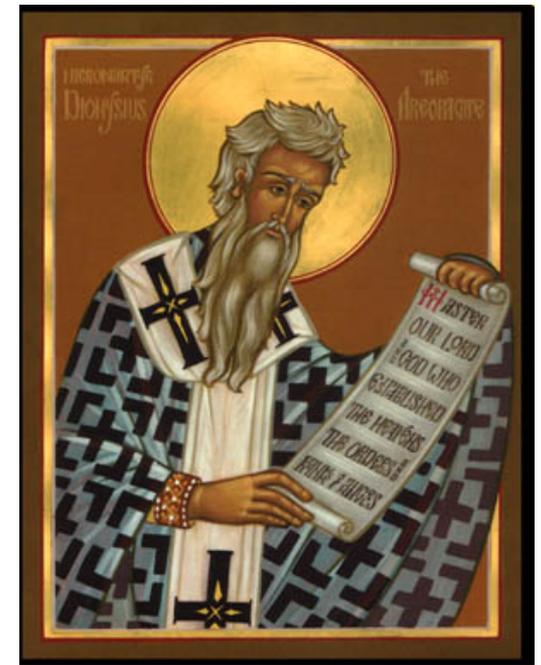
...

# Potential infinity

*synonyms:* bad infinity, apeiron (boundless), indefinitum, syncategorematicum infinitum

## **CHAPTER XIV** from **the Celestial Hierarchy**

(attributed to St. Dionysius the Areopagite)



*What the traditional number of the Angels signifies.*

This also is worthy, I think, of intellectual consideration, that the scriptural tradition respecting the Angels gives their number as thousands of thousands and ten thousand times ten thousand, multiplying and repeating the very highest numbers we have, thus clearly showing that the Orders of the Celestial Beings are innumerable for us; so many are the blessed Hosts of the Supermundane Intelligences, wholly surpassing the feeble and limited range of our material numbers.





## Dante's Paradiso - Canto 29

Translated by Henry F. Cary

No mortal tongue, Nor thought of man,  
hath ever reach'd so far,  
That of these natures he might  
count the tribes. What Daniel of  
their thousands hath reveal'd,  
With finite number, **infinite conceals.**

*Questa natura sì oltre s'ingrada  
in numero, che mai non fu loquela  
né concetto mortal che tanto vada;*

*e se tu guardi quel che si revela  
per Daniel, vedrai che 'n sue migliaia  
determinato numero si cela.*

Illustration to Dante's Divine Comedy  
by Gustave Doré. Plate 34 -  
The "White Rose"

“hold infinity in the palm of your hands”  
*(W.Blake's Auguries of Innocence)*

# Actual infinity

*syn.* positive infinity, proper infinity, kategorematices infinitum

# Actual infinity

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- Absolute infinity – “in Deo”

# Actual infinity

*syn.* positive infinity, proper infinity, kategorematische infinitum

- Absolute infinity – “in Deo”
- Created infinity – the transfinite

# Actual infinity

*syn.* positive infinity, proper infinity, kategorematiche infinitum

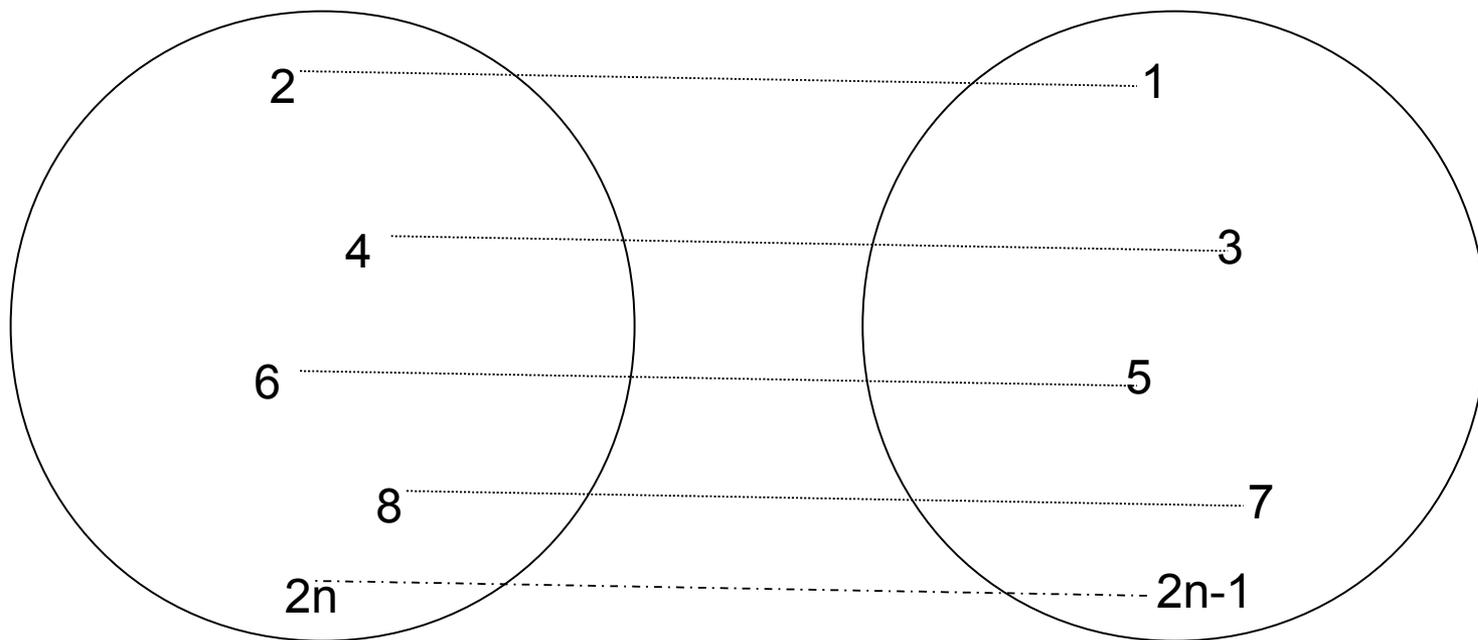
- Absolute infinity – “in Deo”
- Created infinity – the transfinite
  
- As a mathematical magnitude – a symbol

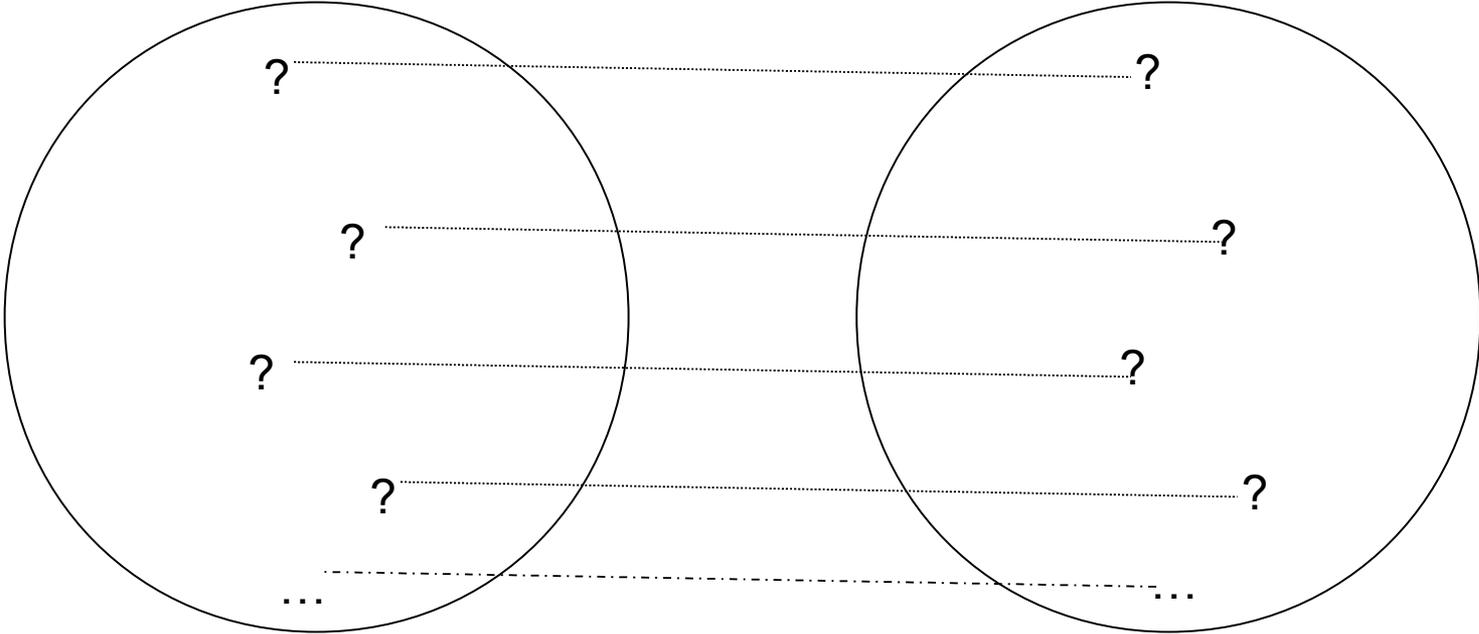


**Transfinite numbers**

*'How many' even numbers are there?*

*'How many' odd numbers are there?*





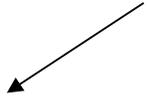
*'How many' even numbers are there?*

2,4,6,8,10,12,...

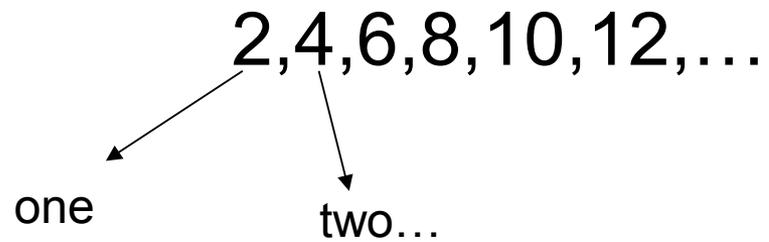
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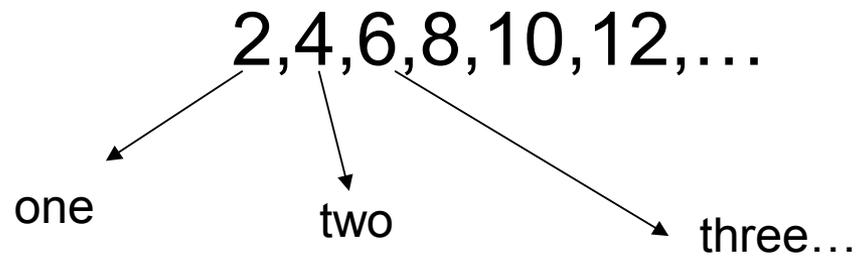
one...

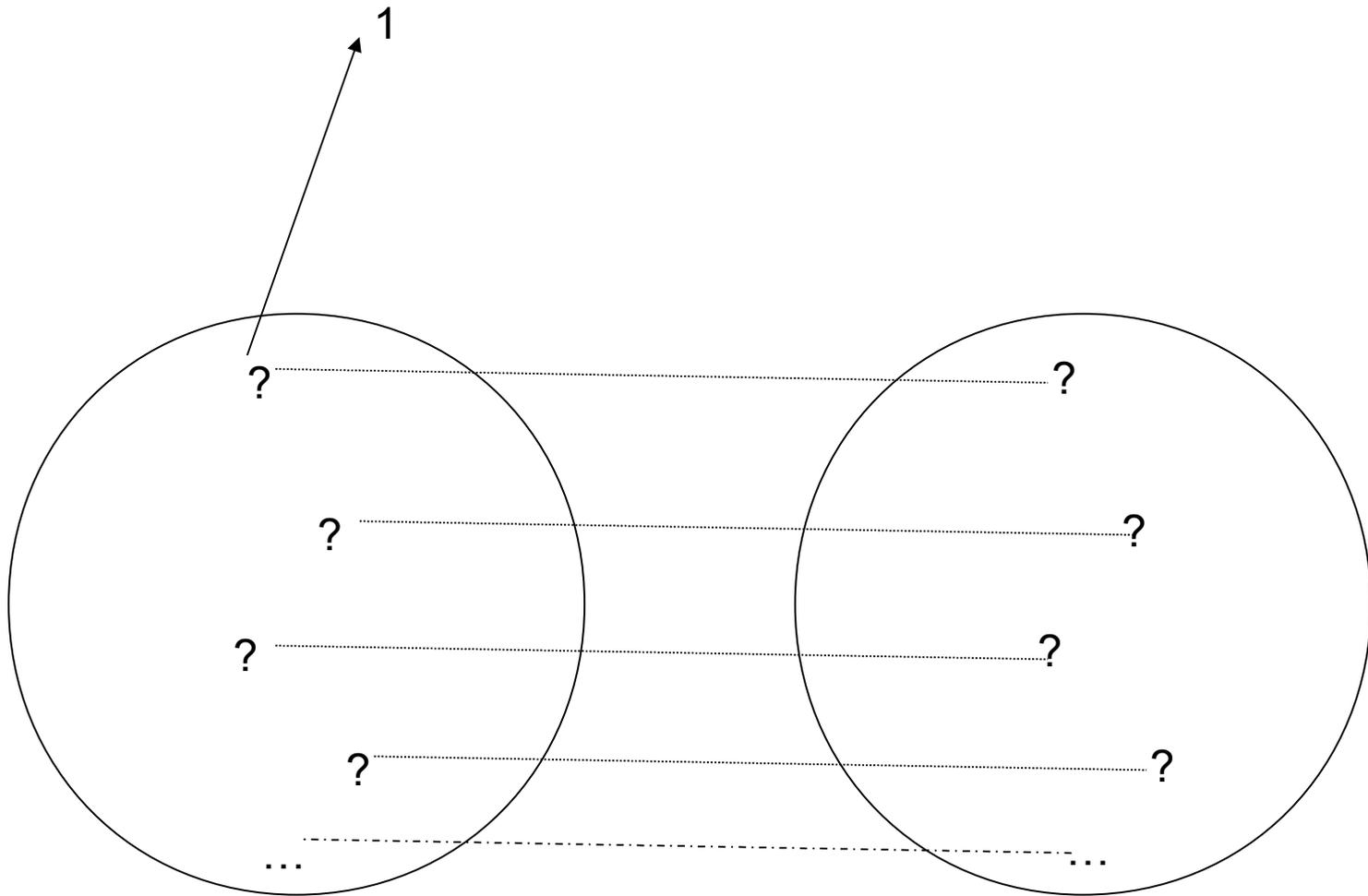


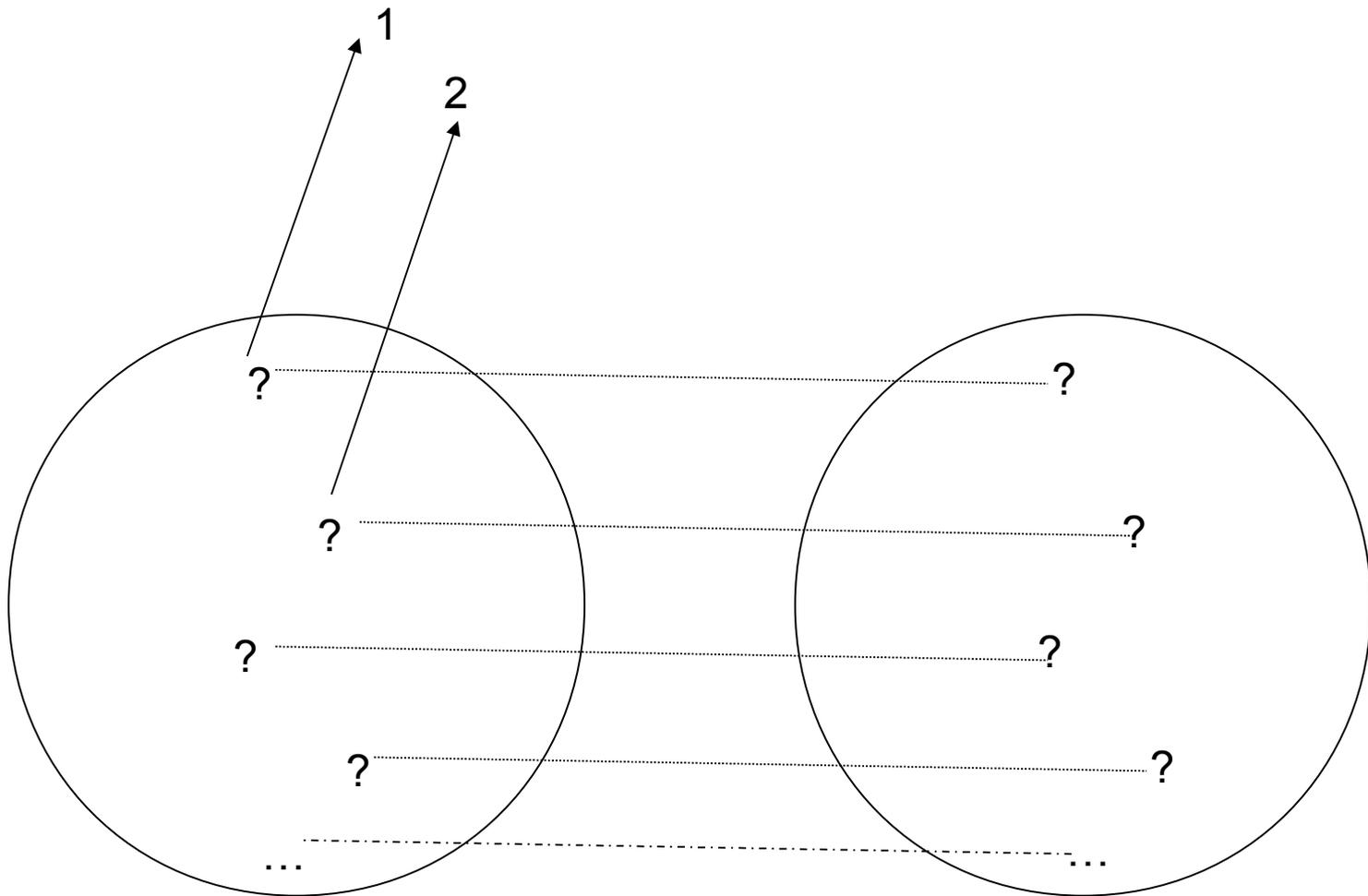
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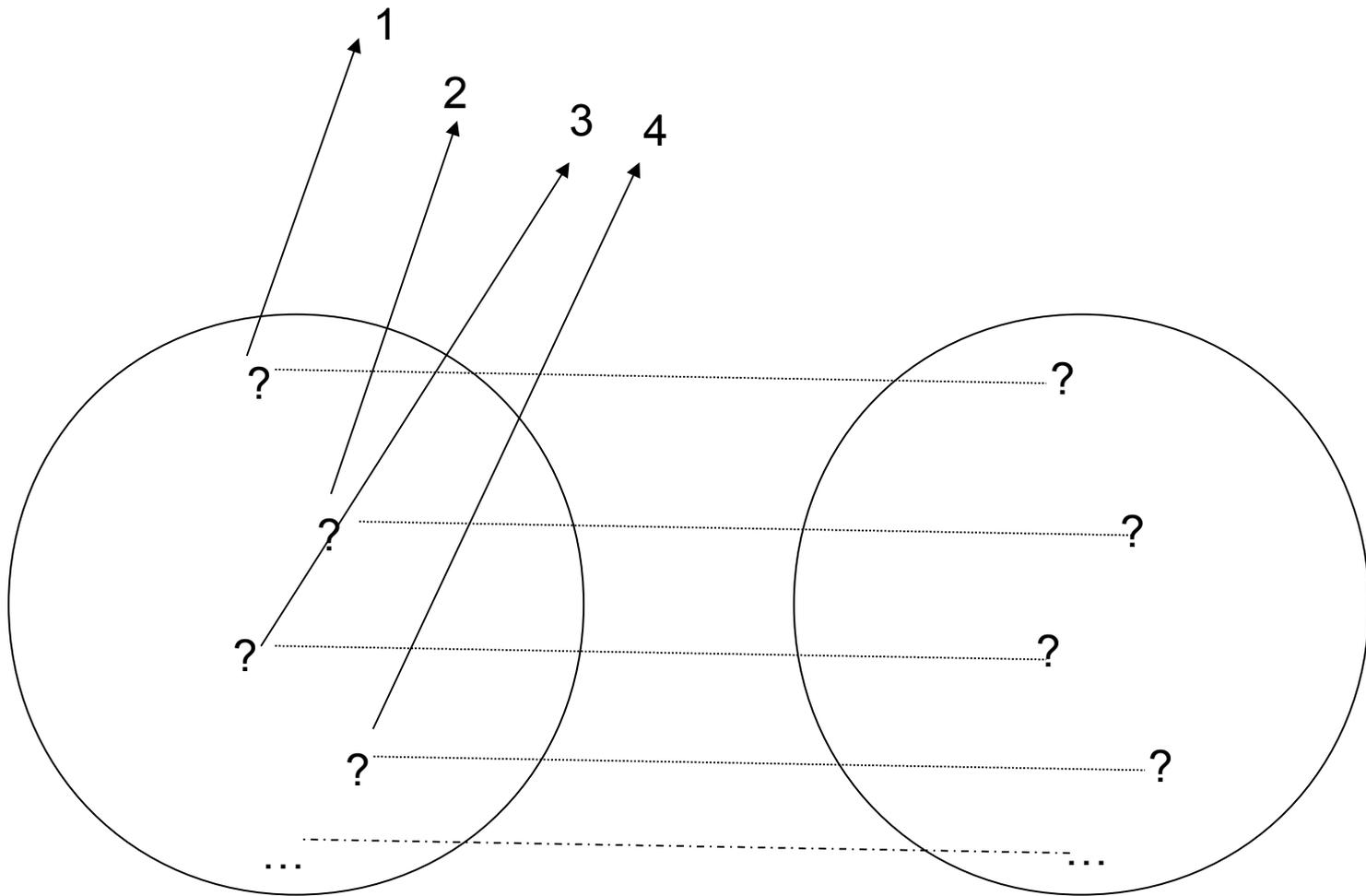


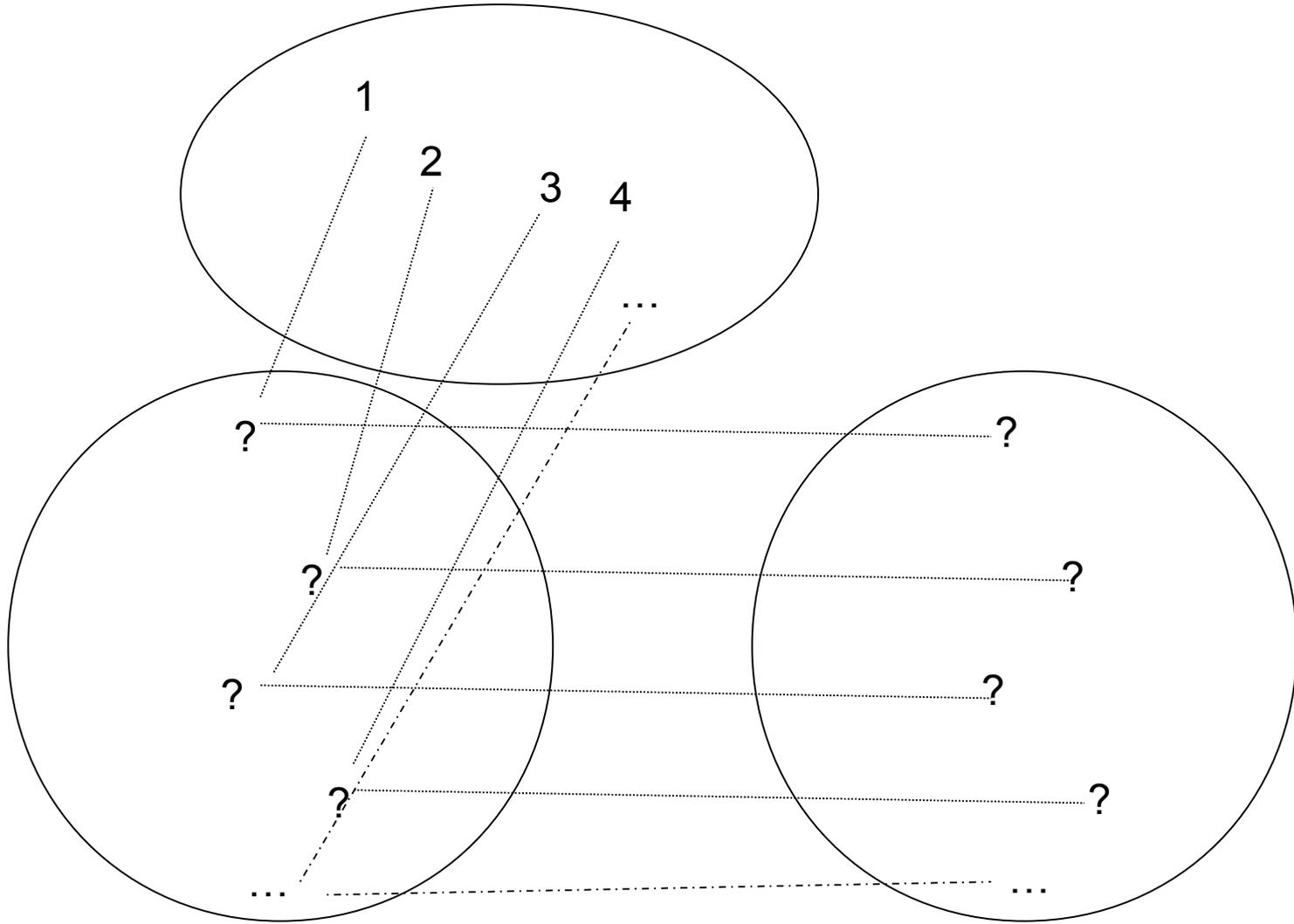
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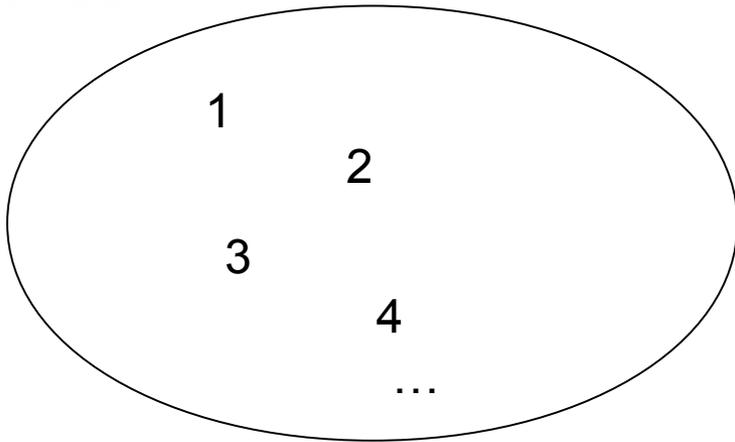


Are the natural numbers more than even numbers?

**No, their (transfinite) number is the same.**

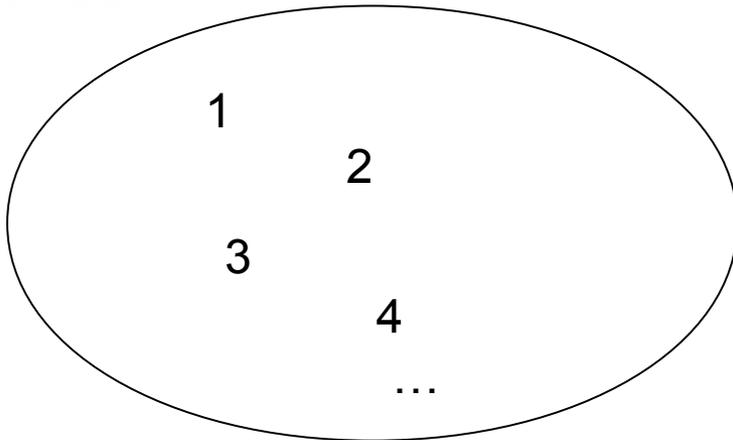
*No*

~~No~~

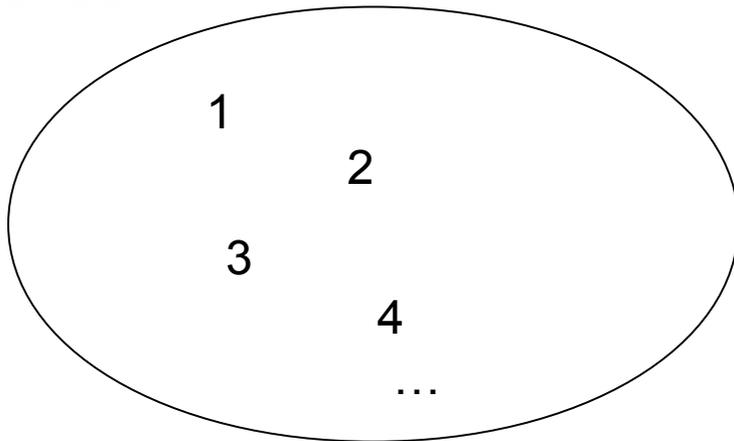


• Absolute infinity?

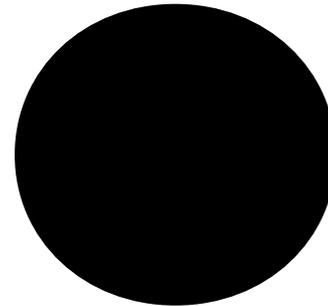
~~No~~



No

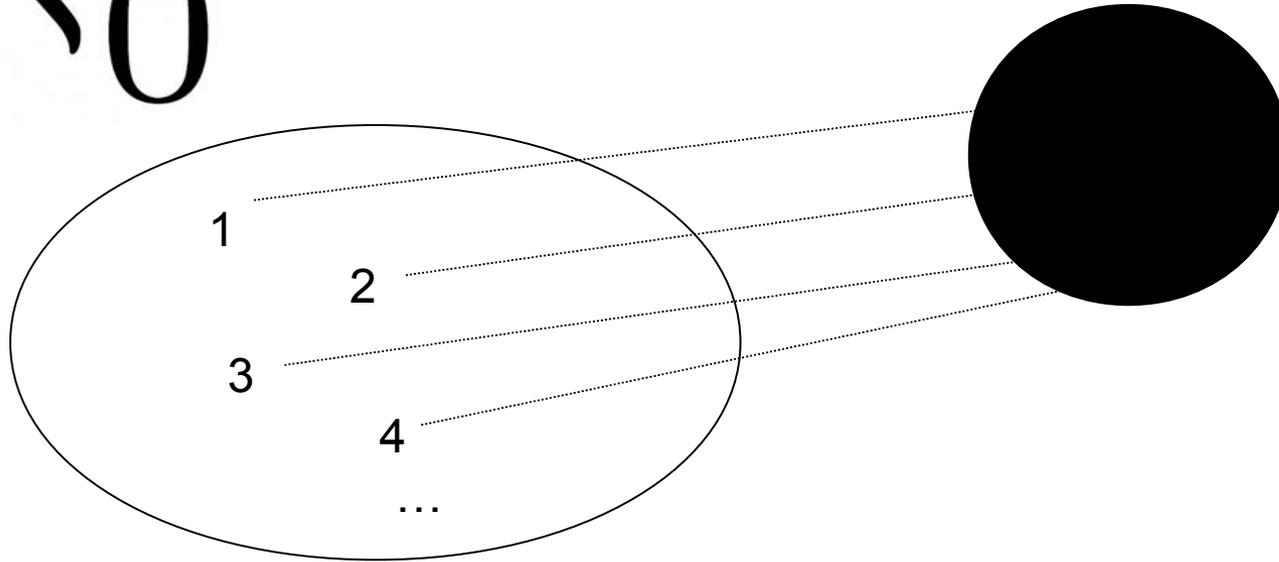


SET OF POINTS INSIDE THE CIRCLE



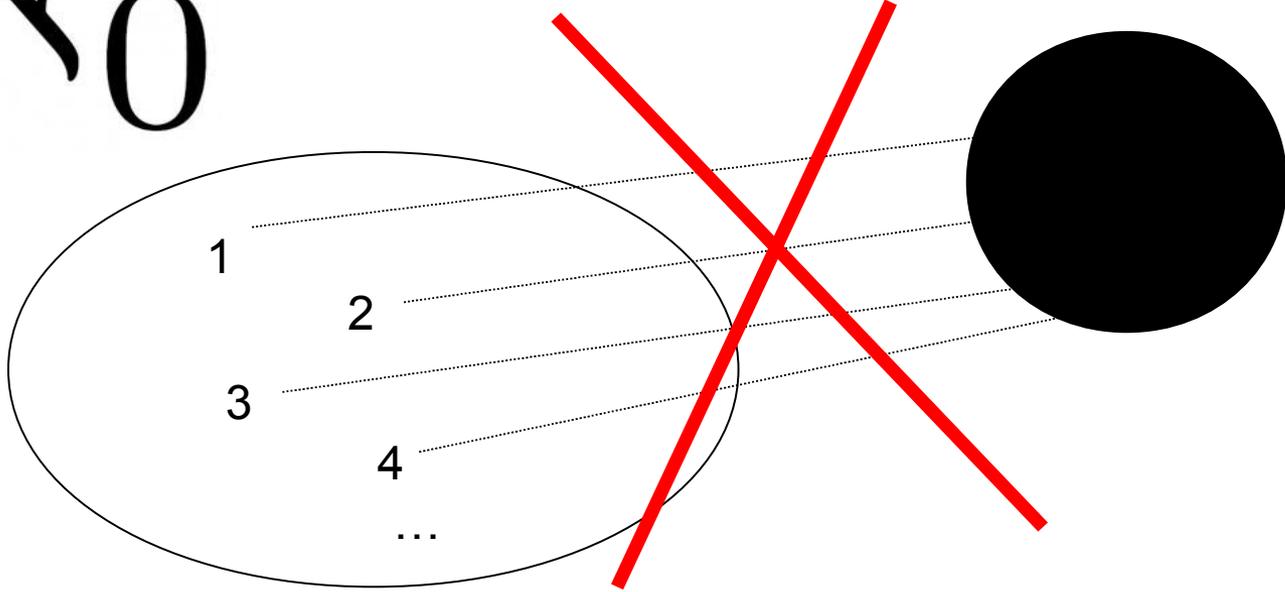
~~No~~

SET OF POINTS INSIDE THE CIRCLE



No

SET OF POINTS INSIDE THE CIRCLE



The number of points inside the circle is **GREATER THAN**  $\aleph_0$



?, ?, ? , ? ...

1,2,3,4,...

?, ?, ? , ? ...

1,2,3,4,...

?, ?, ?, ? ...

This is a way of ordering infinite ( $\aleph_0$ ) elements

1,2,3,4,...

?,?,?,?...

This is a way of ordering infinite ( $\aleph_0$ ) elements

**= an ordinal number!**

1,2,3,4,...

?,?,?,?...

This is a way of ordering infinite ( $\aleph_0$ ) elements

**= an ordinal number, called  $\omega$**

1,2,3,4,...

?,?,?,?...

This is a way of ordering infinite ( $\aleph_0$ ) elements

**= an ordinal number, called  $\omega$**

This is not the only one! E.g.,  $\omega+1, \omega \cdot 2, \omega \cdot 3, \omega^\omega, \omega^{\omega^\omega}$

Re-arrange them like this

2,4,6,...,1,3,5,...

All the even natural numbers and, after them, all the odd numbers

Re-arrange them like this

2,4,6,...,1,3,5,...

All the even natural numbers and, after them, all the odd numbers

another ordinal number, called  $\omega \cdot 2$

"... I realize that in this undertaking I place myself in a certain opposition to views widely held concerning the mathematical infinite and to opinions frequently defended on the nature of numbers".

G.Cantor

"The essence of mathematics  
lies entirely  
in its freedom"

G.Cantor

"The essence of mathematics  
lies entirely  
in its freedom"

G.Cantor

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This defective presentation has been inspired by  
the outstanding works of P.A. Florenskij  
see e.g.

*"On Symbols of the Infinite (An essay on the ideas of G. Cantor)"*.